Special functions and their applications
Fonctions spéciales et leurs applications
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JOHN CAMPBELL, York University
New techniques for evaluating series involving squared central binomial coefficients and harmonic-type numbers

Recently, there has been much interest in the evaluation of series containing expressions such as $\binom{2n}{n}^2 H_n$ and $\binom{2n}{n}^2 H_{2n}$ as factors in the summand for $n \in \mathbb{N}$, letting $H_m = \psi_0(m+1) + \gamma$ denote the harmonic number function, where ψ_0 denotes the digamma function. A variety of integration methods have been introduced recently for symbolically computing such series, which often have interesting evaluations involving $\frac{1}{\pi}$ that are reminiscent of Ramanujan's series for $\frac{1}{\pi}$; this has led to new areas of research investigating connections between Fourier-Legendre theory, the complete elliptic integrals, and the evaluation of binomial-harmonic sums. In this talk, we describe some of the recent advances concerning these new subjects of research.

KARL DILCHER, Dalhousie University
Solutions of certain polynomial Diophantine equations

For each integer $n \geq 1$ we consider the unique polynomials $P, Q \in \mathbb{Q}[x]$ of smallest degree n that are solutions of the equation $P(x)x^{n+1} + Q(x)(x+1)^{n+1} = 1$. We derive numerous properties of these polynomials, including explicit expansions, differential equations, recurrence relations, generating functions, discriminants, irreducibility results, and their zero distribution. We also consider some related polynomials and their properties.

(Joint work with Maciej Ulas, Jagiellonian University).

MOURAD ISMAIL, University of Central Florida
Turning point theory for q-difference equations

We develop a q-difference equation approach to study turning points of q-difference equations. The motivation is to develop Plancherel-Rotach asymptotics of q-orthogonal polynomials. Our method for q-difference equations is an analogue to the turning point problem for Hermite differential equations. This work is joint with Chun-Kong Law from National Sun Yat Sen University in Taiwan.

ALEXEY KUZNETSOV, York University
Generalising Lagrange inversion theorem

Lagrange inversion theorem gives the coefficients of Taylor series expansion of the inverse of an analytic function. In this talk I will discuss some generalisations of Lagrange inversion theorem that apply to Dirichlet series or, more generally, to functions that are Laplace transforms of certain measures.

RICHARD MCINTOSH, University of Regina
On the Universal Mock Theta Function g_2 and Zwegers' μ-function

S.-Y. Kang discovered a formula expressing the universal mock theta function g_2 in terms of Zwegers' μ-function and a theta quotient. By modifying the elliptic variables in μ the theta quotient can be removed from Kang's formula. We also obtain a formula expressing μ in terms of g_2, proving that μ is not more general than g_2, even though it has one more elliptic variable.

ROBERT MILSON, Dalhousie University
Toward the classification of Exceptional Orthogonal Polynomials

Exceptional Orthogonal Polynomials are orthogonal polynomial families that arise as solutions for second-order eigenvalue problems. They generalize the classical families of Hermite, Laguerre, and Jacobi in that they allow for polynomial sequences with a finite number of missing degrees. The fundamental technique for constructing such objects is the Darboux transformation, which "dresses" a classical operator to obtain orthogonal polynomials with a finite number of exceptional degrees. Thanks to a foundational theorem that asserts that all exceptional orthogonal polynomials arise in precisely this fashion, it is now possible to envisage a complete classification of exceptional orthogonal operators and their attendant operators. In my talk I will describe the essential components of this programme and highlight the technical challenges that must be overcome en route to classification.

ALEXANDRU NICA, University of Waterloo
A central limit theorem for the star-generators of the infinite symmetric group

I will present a result with flavour of Central Limit Theorem which takes place in the framework of the infinite symmetric group S_∞, and refers to a character of S_∞ parametrized by a positive integer d. The limit law appearing in this theorem can be described by using Hermite functions, due to its connection with the empirical eigenvalue distribution of a Gaussian Hermitian (GUE) random matrix of size $d \times d$. This is joint work with Claus Koestler, arXiv:1807.05633.

MICHAEL RUBINSTEIN, University of Waterloo
Asymptotics of divisor sums in short intervals, and a Painleve V equation

We discuss the piecewise polynomial functions $\gamma_k(c)$ that appear in the asymptotics of averages of the k-th divisor sum in short intervals, first studied by Keating, Rodgers, Roditty-Gershon, and Rudnick. Specifically, we express these polynomials as the inverse Fourier transform of a Hankel determinant that satisfies a Painlevé V equation. We prove that $\gamma_k(c)$ is very smooth at its transition points, and also determine the asymptotics of $\gamma_k(c)$ in a large neighbourhood of $k = c/2$.

LUC VINET, CRM, Université de Montréal
Generalized Heun operators

A quick survey of the recent advances in the theory of Heun operators will be offered. The characterization of Heun operators associated to families of orthogonal polynomials will be presented.