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The Slow Dynamics of Spot Patterns for Reaction-Diffusion Systems on the Sphere

In the singularly perturbed limit corresponding to large a diffusivity ratio between two components in a reaction-diffusion (RD) system, many such systems admit quasi-equilibrium spot patterns, where the solution concentrates at a discrete set of points in the domain. In this context, we derive and study the differential algebraic equation (DAE) that characterizes the slow dynamics for such spot patterns for the Brusselator RD model on the surface of a sphere. Asymptotic and numerical solutions are presented for the system governing the spot strengths, and we describe the complex bifurcation structure and demonstrate the occurrence of imperfection sensitivity due to higher order effects. Localized spot patterns can undergo a fast time instability and we derive the conditions for this phenomena, which depend on the spatial configuration of the spots and the parameters in the system. In the absence of these instabilities, our numerical solutions of the DAE system for $N = 2$ to $N = 10$ spots suggest a large basin of attraction to a small set of possible steady-state configurations. We discuss the connections between our results and the study of point vortices on the sphere, as well as the problem of determining a set of elliptic Fekete points, which correspond to globally minimizing the discrete logarithmic energy for N points on the sphere.