## Geometric Analysis and Mathematical Relativity
## Analyse géométrique et relativité mathématique

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*Polyhomogeneity of metrics compatible with a Lie structure at infinity along the Ricci flow*

Along the Ricci flow, we study the polyhomogeneity of complete Riemannian metrics endowed with "a Lie structure fibred at infinity", that is, a class of Lie structures at infinity that induce in a precise way a fibre bundle structure on a certain compactification by a manifold with corners. When the compactification is a manifold with boundary, this class of metrics contains, in particular, the b-metrics of Melrose, the fibred boundary metrics of Melrose and Mazzeo and the edge metrics of Mazzeo. Our main result consists in showing that the polyhomogeneity of the metrics compatible with a Lie structure fibered at infinity is locally preserved by the Ricci-DeTurck flow. If the initial metric is asymptotically Einstein, the polyhomogeneity of the metrics solutions is obtained as long as the flow exists. Moreover, if the initial metric is "smooth up to the boundary", then it will be also preserved by the normalized Ricci flow and the Ricci-DeTurck flow.

**ERIC CHEN**, UC Santa Barbara

*Integral pinching for the Ricci flow on asymptotically flat manifolds*

There are many curvature pinching results for the Ricci flow in compact settings, from Hamilton's initial work to work of Brendle–Schoen and others. I will discuss curvature pinching in a noncompact setting: the Ricci flow starting from an asymptotically flat manifold with integral curvature sufficiently pinched exists for all positive times and converges to flat space.

**GRAHAM COX**, Memorial University

*Blowup solutions of Jang's equation near a spacetime singularity*

Jang's equation is a semilinear elliptic equation defined on an initial data set. It was shown by Schoen and Yau that the (non)existence of global solutions is closely related to the presence of apparent horizons, which are quasi-local analogues of black hole boundaries. As a result, Jang's equation can be used to prove the existence of apparent horizons by imposing appropriate geometric conditions on the initial data set. These proofs proceed by contradiction: one assumes there is a global solution, then proves that its existence is not compatible with the given geometric assumptions.

In this talk I will outline a constructive approach to proving the existence of apparent horizons. In particular, I will consider a distinguished family of spacelike hypersurfaces in the maximally extended Schwarzschild spacetime, and prove that Jang's equation admits no global solutions once the hypersurfaces become sufficiently close to the r=0 singularity. This suggests a general strategy for relating spacetime singularities to apparent horizons. This is joint work with Amir Aazami.

**SHUBHAM DWIVEDI**, University of Waterloo

*Flows of $\mathrm{G}_2$ structures.*

We will discuss a family of flows of $\mathrm{G}_2$ structures on seven dimensional Riemannian manifolds. We will prove short-time existence and uniqueness of solution to the flows. Time permitting, we will discuss a special case called the "isometric flow" of $\mathrm{G}_2$ structures, in detail. This is a joint work with Panagiotis Gianniotis and Spiro Karigiannis.

**AILANA FRASER**, University of British Columbia
*Some results on higher eigenvalue optimization*

We will discuss some recent results concerning the optimization higher eigenvalues of manifolds with boundary, both in two and higher dimensional cases. This talk is based on joint work with P. Sargent and R. Schoen.

**GREG GALLOWAY**, University of Miami
*Topology and singularities in cosmological spacetimes obeying the null energy condition*

The relationship between the topology of spacetime and the occurrence of singularities (causal geodesic incompleteness) is a topic of long-standing interest. In this talk we focus on the cosmological setting: We consider globally hyperbolic spacetimes with compact Cauchy surfaces under assumptions compatible with the presence of a positive cosmological constant. More specifically, for 3+1 dimensional spacetimes which satisfy the null energy condition and contain a future expanding compact Cauchy surface, we establish a precise connection between the topology of the Cauchy surfaces and the occurrence of past singularities. In addition to (a refinement of) the Penrose singularity theorem, the proof makes use of certain fundamental existence results for minimal surfaces and of some recent advances in the topology of 3-manifolds. This talk is based on joint work with Eric Ling.

**MELANIE GRAF**, University of Washington
*Generalized cones as Lorentzian length spaces*

Smooth Lorentzian warped products of the form $I \times_f (M, g)$, where $(M, g)$ is a Riemannian manifold and $f$ is a positive smooth function on an intervall $I$, are important examples of spacetimes: They contain well-known physical models (such as the FLRW spacetimes) and admit a very simple description of causal curves and geodesics. In this talk we will examine what happens if one replaces the Riemannian manifold with a length space: We shall see that for these *generalized cones* one still has a natural notion of causal curves and their length and hence also of the causality relations, turning them into Lorentzian length spaces. Moreover, synthetic timelike curvature bounds of such generalized cones are directly related to Alexandrov curvature bounds of the length space and convexity/concavity properties of $f$. Finally, we will formulate (and sketch the proof of) singularity theorems for these spaces. This is joint work with S. B. Alexander, M. Kunzinger and C. Sämann.

**CHRISTINE GUENTHER**, Pacific University
*Convergence Stability of the Ricci Flow*

We define the principle of *convergence stability* for geometric flows, which says that if a solution exists for all time and converges to a stable fixed point, then solutions that start at nearby geometries also converge to fixed points. In particular, convergence results obtained for symmetrical spaces can be extended to geometries without symmetries. We show convergence stability of the Ricci flow on compact manifolds, by first using analytic semigroup methods to prove long time continuous dependence of solutions on initial conditions, and then invoking known stability results at flat (or hyperbolic) fixed points. We further present our current work to generalize these results to asymptotically hyperbolic manifolds. This is joint work with Eric Bahuaud (Seattle University) and James Isenberg (University of Oregon).

**ROBERT HASLHOFER**, University of Toronto
*Ancient mean curvature flows and the mean convex neighborhood conjecture*

I will explain our recent proof of the mean convex neighborhood conjecture. The key is a classification result for ancient asymptotically cylindrical mean curvature flows. The 2-dimensional case is joint work with Choi and Hershkovits, and the higher-dimensional case is joint with Choi, Hershkovits and White.

**MOHAMMAD IVAKI**, University of Toronto
*Mean curvature flow with free boundary*

Non-collapsing plays a fundamental role in the analysis of mean curvature flow. In this talk, I will discuss how Brian White's measure theoretic approach can be generalized to yield the non-collapsing for mean curvature flow with free boundary, provided the barrier is mean convex. This is joint work with N. Edelen, R. Haslhofer and J. Zhu.

**CHRISTIAN KETTERER**, University of Toronto
*RCD metric measure spaces with Alexandrov upper curvature bounds*

In this talk I will present a structure theory for RCD metric measure spaces with curvature bounded from above in the sense of Alexandrov. In particular, the space is a topological manifold with boundary whose interior is the set of regular points. This is a joint work with Vitali Kapovitch and Martin Kell.

**MARCUS KHURI**, Stony Brook University
*Geometric Inequalities for Quasi-Local Masses*

We will describe lower bounds for quasi-local masses in terms of charge, angular momentum, and horizon area. In particular we treat three quasi-local masses based on a Hamiltonian approach, namely the Brown-York, Liu-Yau, and Wang-Yau masses. The geometric inequalities are motivated by analogous results for the ADM mass. They may be interpreted as localized versions of these inequalities, and are also closely tied to the conjectured Bekenstein bounds for entropy of macroscopic bodies. In addition, we give a new proof of the positivity property for the Wang-Yau mass which is used to remove the spin condition in higher dimensions. Furthermore, we generalize a recent result of Lu and Miao to obtain a localized version of the Penrose inequality for the static Wang-Yau mass. This is joint work with A. Alaee and S.-T. Yau.

**HARI KUNDURI**, Department of Mathematics and Statistics, Memorial University
*Divergence identities for stationary vacuum black holes*

I will sketch the derivation of new identities relating the geometric invariants of five-dimensional, asymptotically flat, stationary and biaxisymmetric vacuum black hole solutions. In addition to the usual physical charges (e.g. mass, angular momenta) these identities include contributions from the topology of the spacetime. The proof employs the harmonic map formulation of the vacuum Einstein equations for solutions with these symmetries.

**ROBERT MCCANN**, University of Toronto
*Entropic convexity and the Einstein equation for gravity*

On a Riemannian manifold, lower Ricci curvature bounds are known to be characterized by geodesic convexity properties of various entropies with respect to the Kantorovich-Rubinstein-Wasserstein square distance from optimal transportation. These notions also make sense in a (nonsmooth) metric measure setting, where they have found powerful applications. In this talk I describe the development of an analogous theory for lower Ricci curvature bounds in timelike directions on a Lorentzian manifold. In particular, by lifting fractional powers of the Lorentz distance (a.k.a. time separation function) to probability measures on spacetime, I show the strong energy condition of Penrose is equivalent to geodesic concavity of the Boltzmann-Shannon entropy there. Parallel work of Mondino and Suhr gives also the complementary upper bound, hence a reformulation of the Einstein field equations of general relativity in terms of the convexity properties of the entropy.

See preprint at http://www.math.toronto.edu/mccann/papers/GRO.pdf

**FRÉDÉRIC ROCHON**, UQÀM
*Geometrically finite Poincaré-Einstein metrics*

After reviewing the notion of geometrically finite hyperbolic metrics, I will explain how perturbing their conformal infinity yields new examples of complete Einstein metrics via the inverse function theorem. This is a joint work with Eric Bahuaud.

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**CLEMENS SAEMANN**, University of Toronto
*An introduction to Lorentzian length spaces*

We introduce an analogue of the theory of length spaces into the setting of Lorentzian geometry and causality theory. The role of the metric is taken over by the time separation function, in terms of which all basic notions are formulated. In this way we recover many fundamental results in greater generality, while at the same time clarifying the minimal requirements for and the interdependence of the basic building blocks of the theory. A main focus of this work is the introduction of synthetic curvature bounds, akin to the theory of Alexandrov and CAT(k)-spaces, based on triangle comparison. Applications include Lorentzian manifolds with metrics of low regularity, closed cone structures, and warped products of a line with a (Riemannian) length space. Moreover, we give an application to the low regularity (in)-extendibility of spacetimes and show that inextendibility is related to a (synthetic) curvature blow-up. In a follow-up talk Melanie Graf will detail the application to *generalized cones*, i.e., Lorentzian warped products with one-dimensional base, and their causality, curvature and also discuss singularity theorems in this setting.

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**JEROME VÉTOIS**, McGill University
*Blowing-up solutions to low-dimensional scalar curvature-type equations*

In this talk, we will consider the question of existence of positive blowing-up solutions to a class of scalar curvature-type equations on a closed Riemannian manifold. A result of Olivier Druet provides necessary conditions for the existence of blowing-up solutions whose energy is a priori bounded. We will present new results showing the optimality of Druet's conditions. We will discuss in particular the low-dimensional case, where the mass of the manifold plays a major role. This is a joint work with Frédéric Robert (Université de Lorraine).