Chord Diagrams Everywhere
(Org: **Marcel Golz** and/et **Karen Yeats** (Waterloo))

SUSAMA AGARWALA, Johns Hopkins Applied Physics Lab
Chord Diagrams in N=4 SYM field theory

In this talk, I introduce diagrams representing particle interactions in the quantum field theory N=4 SYM called Wilson loop diagrams. I give a condition for when two diagrams are geometrically equivalent. These diagrams give rise to chord diagrams, which help enumerate the number of other Wilson loop diagrams exist in each equivalence class. It is still an open question as to how many different equivalence classes exist.

MAX ALEKSEYEV, George Washington University
Unlabeled Motzkin numbers

The n-th Motzkin number represents the number of configurations of nonintersecting chords connecting n labeled points on a circle. Assuming that the points are unlabeled and equally spaced along the circle, we compute the number of nonisomorphic chord configurations. We also compute the number of configurations that are not symmetric w.r.t. rotation by any nontrivial angle.

DROR BAR-NATAN, University of Toronto, Mathematics
Chord Diagrams, Knots, and Lie Algebras

This will be a service talk on ancient material - I will briefly describe how the exact same type of chord diagrams (and relations between them) occur in a natural way in both knot theory and in the theory of Lie algebras. See http://drorbn.net/t19.

MICHAEL BORINSKY, Nikhef
Constrained graph counting

Many interesting topological invariants can be expressed as sums over sets of graphs. I will report on a recent joint work with Karen Vogtmann in which such techniques were applied to proof a conjecture on the Euler characteristic of $\mathrm{Out}(F_n)$. The combinatorial part of the proof is a generalized graph counting lemma, which enables the enumeration of constrained sets of graphs. I will illustrate this key step and highlight the appearance of chord diagrams.

JAMES DAVIES, University of Waterloo
Circle graphs are polynomially χ-bounded

A class of graphs is χ-bounded if there exists a function that bounds the chromatic number of a graph in terms of the size of its largest complete subgraph. We prove that circle graphs have a polynomial χ-bounding function.

Joint work with Rose McCarty.

NICK EARLY, Perimeter Institute for Theoretical Physics
From weakly separated collections to matroid subdivisions

We study arrangements of slightly skewed tropical hyperplanes, called blades, on the vertices of a hypersimplex $\Delta_{k,n}$. We reformulate the condition under which such an arrangement induces a matroid (in fact a positroid) subdivision as the requirement that the collection of vertices must satisfy pairwise a certain non-crossing condition on a chord diagram: pairs of vertices should be weakly separated. This work is motivated in part by recent joint work together with Cachazo, Guevara and Mizera, where a formulation was proposed for quantum field theory over $\mathbb{CP}^{k-1}$.

This talk is based on preprints 1910.11522 and 1810.03246.

JO ELLIS-MONAGHAN, Carnegie Mellon/Saint Michael's College

Generating Graph Dualities from Chord Diagrams

In joint work with Lowell Abrams (George Washington University) on twisted duality, we develop tools to identify and generate new surface embeddings of graphs with various forms of self-duality including geometric duality, Petrie duality, Wilson duality, and both forms of triality (which is like duality, but of order three instead of two). Previous work typically focused on regular maps (special, highly symmetric, embedded graphs), but the methods presented here apply to general embedded graphs. In contrast to Wilson's very large self-trial map of type $\{9,9\}_9$ we show that there are self-trial graphs on as few as three edges. We reduce the search for graphs with some form of self-duality or self-triality to the study of orientable one-vertex ribbon graphs, i.e. chord diagrams. We use the chord diagrams as the basis a fast algorithm that will find all graphs with any of the various forms of self-duality or self-triality in the orbit of a graph that is isomorphic to any twisted dual of itself.

MARCEL GOLZ, University of Waterloo

Chord diagrams in perturbative quantum electrodynamics

Feynman graphs in quantum electrodynamics are essentially chord diagrams with photons taking the role of chords attached to lines or cycles given by electrons. I will discuss some of the ways combinatorial methods involving chord diagrams simplify the associated Feynman integrals and in particular the computation of traces and contractions of Dirac matrix products.

CHRISTINE HEITSCH, Georgia Institute of Technology

Meanders and RNA Folding

A closed meander of order n is a non-self-intersecting closed curve in the plane which crosses a horizontal line at $2n$ points. Meanders occur in a variety of settings from combinatorial models of polymer folding to the Temperley-Lieb algebra, yet the exact enumeration problem remains open. Building on results for plane trees and noncrossing partitions motivated by the biology of RNA folding, we prove that meanders are connected under appropriately defined local move transformations. The resulting meander graphs have some interesting characteristics, which may lead to new counting insights. As we will explain, meanders also relate to the challenging biomathematical problem of comparing different possible folds for an RNA sequence.

ALI MAHMOUD, University of Waterloo

On the Asymptotics of Connected Chord Diagrams

We pursue a combinatorial interpretation for expressions that appear in the asymptotic expansion of C_n, the number of connected chord diagrams on n chords. The main outcome presented here is a new combinatorial interpretation for entry A088221 of the OEIS. We will show that A088221 surprisingly counts pairs of connected chord diagrams (allowing empty diagrams). This question arose from a more applied context, namely, from quantum field theory where connected chord diagrams are used in describing solutions to the Dyson-Schwinger equations. The problem considered here come as a small outgrowth of a larger ongoing program aiming to replace the sometimes ill-defined analytic understanding of quantum field theory with a discrete combinatorial understanding that represents itself in a way that is more elementary, yet more robust.

ROSE MCCARTY, University of Waterloo

Vertex-Minors and Circle Graphs

A vertex-minor of a graph G is a graph which can be obtained from G by deleting vertices and performing local complementations; locally complementing at a vertex v replaces the induced subgraph on the neighborhood of v by its complement. For circle graphs, locally complementing corresponds to a natural operation on chord diagrams. We discuss the role that circle graphs play in studying vertex-minors. In particular, we discuss a theorem which says that every "non-decomposable" class of graphs contains all circle graphs as vertex-minors. Joint work with Jim Geelen, O-joung Kwon, and Paul Wollan.

LUKAS NABERGALL, University of Waterloo
Connectivity and terminal chords in chord diagrams

Motivated by chord diagram expansions appearing in recent solutions for certain analytic Dyson-Schwinger equations from quantum field theory, we present a bijection between chord diagrams of size $n-1$ and chord diagrams of size n with exactly one terminal chord, a sink vertex in the directed intersection graph of the diagram. We then describe several results towards characterizing the relationship between this bijection and the connectivity of chord diagrams.

DAVID SANKOFF, University of Ottawa
Evolution in a cup

Plants evolve differently from other living forms. True the basic mechanism of DNA mutation from generation to generation is universal, with its substitutions, insertions and deletions in long strings ("chromosomes") of the four bases A,C,G and T. And like other organisms, chromosomes (each considered as a series of a few hundred or thousand distinct genes) may change by inverting segments of any length or by moving segments from one chromosome to another or to a different position on the same chromosome. But plants also evolve by autotetraploidization, duplicating their entire genomes, creating an extra copy of every chromosome and every gene within a single new genome. Or by allotetraploidization, combining two highly similar genomes into one, with two distinct subgenomes. Virtually every existing flowering plant has had one, two or more tetraploidizations in its history. Following such whole genome duplications (or triplications, etc.), the augmented genomes lose most of the extra genes, from one subgenome or the other, over a period of many generations, by a process known as fractionation. The arabica coffee genome was formed by the allotetraploidization of robusta coffee and another species, eugenioides, hundreds of thousands of years ago. A CIRCOS representation of the three species, appropriately arranged chromosome by chromosome in a single circular configuration, with chords connecting homologous chromosomal fragments in two genomes or subgenomes, captures the entire evolutionary history of arabica. In it we can see displayed the robusta-eugenioides speciation many millions of years ago, their more recent allotetraploidization, and the effects of fractionation.