Convexity in Algebraic Geometry and Symplectic Geometry
Convexité en géométrie algébrique et géométrie symplectique
(Org: **Kiumars Kaveh** (University of Pittsburgh) and/et **Sergio Da Silva** (Manitoba))

BALAZS ELEK, University of Toronto
Pizzas and Richardson varieties

When does a toric surface degenerate into a union of Schubert, or Richardson varieties? We will answer this question using linear algebra, with a surprise appearance from the braid group on 3 strands.

LAURA ESCOBAR, Washignton University in St. Louis
Matrix Schubert varieties with a view towards T-varieties

Kazhdan-Lusztig ideals are combinatorially-defined polynomial ideals which are generated by minors of a generic matrix. Each ideal in our class encodes the coordinates and equations for a neighborhood of a Schubert variety at a torus fixed point. Included within this class are the ideals defining matrix Schubert varieties. In this talk, we present combinatorial aspects arising from a natural torus action on Kazhdan-Lusztig ideals. We will describe the complexity of this torus action. This is joint work in progress with Maria Donten-Bury and Irem Portakal.

MEGUMI HARADA, MCMASTER UNIVERSITY
Spherical amoebas and spherical tropicalization

(This is a report on joint work in progress with Johannes Hofscheier and Kiumars Kaveh.)

Let X be a subvariety of the n-dimensional complex torus. It is well-known in tropical geometry that the "amoebas" of X limit to the tropicalization of X. It is natural to ask whether there is a non-abelian analogue of this phenomenon. We take the point of view that an answer should be phrased in the setting of (spherical homogeneous spaces and) spherical varieties, which can be thought of as non-abelian analogues of toric varieties. Indeed, in 2016, Kaveh and Manon defined a "spherical tropicalization map" for spherical homogeneous spaces. Thus we can ask: for a subvariety Y of a spherical homogeneous space, is there a (family of) spherical amoebas which limit to the image of Y under the spherical tropicalization map, i.e. strop(Y)? I will explain the definitions necessary to state the question precisely, and discuss some preliminary results pointing to an answer.

LISA JEFFREY, University of Toronto
The SU(2) commutator map and character varieties

(Joint work with Nan-Kuo Ho, Paul Selick and Eugene Xia)

We study the space of conjugacy classes of representations of the fundamental group of a punctured genus 2 surface into $SU(2)$, with the constraint that the loop around the puncture is sent to $-I$ (minus the identity matrix). In other words $A = M/SU(2)$ where M is the space of representations of π to SU(2) which send the loop around the puncture to $-I$, where π is the fundamental group of a punctured genus 2 surface. We recover the Betti numbers of A (a special case of the results found by Atiyah and Bott in their landmark 1982 paper). In this special case, we recover their result by much more elementary methods: a Mayer-Vietoris sequence using a decomposition of the space as the union of two subspaces, each of which retracts to $\mathcal{T}$, the space of commuting pairs in SU(2). Our main results include a new computation of the cohomology ring of A by elementary methods, and a computation of the cohomology groups of M. We also compute the ring structure of $\mathcal{T}$. We construct a retraction of two open dense subsets of A to $\mathcal{T}$.

YAEL KARSHON, University of Toronto
Bott Canonical Basis?

Together with Jihyeon Jessie Yang, we are resurrecting an old idea of Raoul Bott for using large torus actions to construct canonical bases for unitary representations of compact Lie groups. Our methods are complex analytic; we apply them to families of Bott-Samelson manifolds parametrized by $\mathbb{C}^n$. Our construction requires the vanishing of higher cohomology of sheaves of holomorphic sections of certain line bundles over the total spaces of such families; this vanishing is conjectural, hence the question mark in the title.

ASKOLD KHOVANSKII, University of Toronto
Newton polyhedron and hypersurfaces in toric varieties

With a compact smooth toric variety M and a fixed positive 0-cycle A (a fixed finite set of points equipped with positive integral multiplicities) belonging to the union M^1 of one-dimensional orbits of M one can associate the following problem: *find all hypersurfaces $H \subset M$ such that H does not pass through null-dimensional orbits and the intersection of H with M^1 is the 0-cycle A.*

This problem was solved in the case when $\dim M = 2$ in [1]. Let M_O be the closure in M of an orbit O. Let A_O be the 0-cycle $A \cap M_O$.

Theorem. *The problem has at least one solution H if and only if for each two-dimensional orbit O the problem for the toric surface M_O and the 0-cycles A_O has at least one solution.*

Moreover the intersection of any solution H with the torus $(\mathbb{C}^*)^n$ can be defined by equation $Q = 0$ where Q is a Laurent polynomial whose Newton polyhedron Δ and coefficients at monomials belonging to edges of Δ can be found explicitly and whose coefficients at all other monomials in Δ are arbitrary complex numbers.

References

1. A. Khovanskii. Newton polygons, curves on torus surfaces, and the converse Weil theorem, Russian Math. Surveys 52 (1997), no. 6, 1251-1279.

JEREMY LANE, McMaster University
Big toric charts on regular coadjoint orbits of compact Lie groups

Harada and Kaveh defined dense toric charts on integral coadjoint orbits of compact Lie groups via toric degeneration. In this talk I will describe a new approach to constructing toric charts on coadjoint orbits of compact Lie groups. Although we are not able to construct dense toric charts due to analytical limitations, we are able to construct toric charts on regular coadjoint orbits that are "big" in the sense that they exhaust the symplectic volume of the coadjoint orbit. These big charts are sufficient to extend known results regarding the Gromov width of coadjoint orbits.

This talk is based on collaboration with Anton Alekseev, Benjamin Hoffman, and Yanpeng Li, arXiv:1804.01504, arXiv:1808.06975, and work in progress.

MARTHA PRECUP, Washington University in St. Louis
A generalization of the Springer resolution

The Springer correspondence relates irreducible representations of the symmetric group to a subset of simple perverse sheaves on the nilpotent cone. The Springer resolution $\mu : \widetilde{\mathcal{N}} \to \mathcal{N}$ of the nilpotent cone and its fibers play an essential role in this result. In the 1980's, Lusztig proved that each simple perverse sheaf on the nilpotent cone corresponds to an irreducible representation of a relative Weyl group. This series of results is known as the generalized Springer correspondence.

The focus of this talk will be a map $\psi : \widetilde{\mathcal{M}} \to \mathcal{N}$ defined by Graham. The space $\widetilde{\mathcal{M}}$ is constructed using affine toric varieties, and the rich theory of these varieties can be used to obtain detailed information about $\widetilde{\mathcal{M}}$. We will discuss the relationship between Graham's construction and the generalized Springer correspondence for $SL_n(\mathbb{C})$, and describe the fibers of ψ using the combinatorics of standard tableaux. This talk is based on joint work with William Graham and Amber Russell.

JENNA RAJCHGOT, University of Saskatchewan
A combinatorial proof of the type A quiver component formula

The K-theoretic quiver component formula expresses the K-polynomial of a type A quiver locus as an alternating sum of products of double Grothendieck polynomials. This formula was conjectured by A. Buch and R. Rimányi and later proved by R. Kinser, A. Knutson, and the speaker using geometric methods.

After motivating the study of this and related formulas, I will outline this geometric proof. Then I will explain the ingredients of a new proof of this formula which replaces Gröbner degenerations by combinatorics. This latter work is the outcome of an undergraduate summer research project with Aidan Lindberg.

MATTHEW SATRIANO, University of Waterloo
Lifting Tropical Self-Intersections

Given a plane curve C, we say a tropical divisor D on $\mathrm{Trop}(C)$ is C-realizable if there exists a plane curve C' with $D = \mathrm{Trop}(C \cap C')$ and $\mathrm{Trop}(C) = \mathrm{Trop}(C')$. We prove that the set of C-realizable divisors form a polyhedral complex. Moreover, if the genus of C is at most 1, we give a combinatorial condition guaranteeing realizability of D. This is based on joint work with Yoav Len.

IVAN SOPRUNOV, Cleveland State University
Collections of lattice polytopes with a given mixed volume

Recently Esterov and Gusev showed that the problem of classifying generic sparse polynomial systems which are solvable in radicals reduces to the problem of classifying collections of lattice polytopes of mixed volume up to 4. Given the value of mixed volume m and dimension d, there exist only finitely many collections $(P_1, \ldots, P_d)$ of d-dimensional lattice polytopes of mixed volume m, up to unimodular transformations. One reason for this is that the volume of $P_1 + \cdots + P_d$ is bounded above by $O(m^{2^d})$, as follows by direct application of the Aleksandrov-Fenchel inequality. We employ more relations between mixed volumes to improve the bound to $O(m^d)$, which is asymptotically sharp. We also produce a complete classification of inclusion-maximal triples of lattice polytopes in $\mathbb{R}^3$ of mixed volume up to 4. This is joint work with Gennadiy Averkov and Christopher Borger.

JENYA SOPRUNOVA, Kent State University
Lattice Size of Polytopes

The lattice size $\mathrm{ls}(P)$ of a lattice polytope P is the smallest integer l such that after a unimodular transformation P fits into the l-dilate of the standard simplex. The lattice size was introduced and studied by Schicho, Castryck, and Cools in the context of simplification of a parametrization of an algebraic surface.

We explain the connection of the lattice size of P to its successive minima and show that in the case of polygons a reduced basis computes the lattice size, which leads to a very fast algorithm for computing $\mathrm{ls}(P)$.

We also provide an algorithm for finding a reduced basis is dimension 3, analyze its complexity, and explain the connection of the reduced basis to the successive minima. Although it is not true in dimension 3 that a reduced basis computes the lattice size, we show that for empty lattice tetrahedra there exists a reduced lattice basis that computes the lattice size. This is joint work with A. Harrison, A. Alajmi, and P. Tierney.