LISA JEFFREY, University of Toronto

The SU(2) commutator map and character varieties

(Joint work with Nan-Kuo Ho, Paul Selick and Eugene Xia)

We study the space of conjugacy classes of representations of the fundamental group of a punctured genus 2 surface into $SU(2)$, with the constraint that the loop around the puncture is sent to $-I$ (minus the identity matrix). In other words $A = M/SU(2)$ where M is the space of representations of π to SU(2) which send the loop around the puncture to $-I$, where π is the fundamental group of a punctured genus 2 surface. We recover the Betti numbers of A (a special case of the results found by Atiyah and Bott in their landmark 1982 paper). In this special case, we recover their result by much more elementary methods: a Mayer-Vietoris sequence using a decomposition of the space as the union of two subspaces, each of which retracts to $\mathcal{T}$, the space of commuting pairs in SU(2). Our main results include a new computation of the cohomology ring of A by elementary methods, and a computation of the cohomology groups of M. We also compute the ring structure of $\mathcal{T}$. We construct a retraction of two open dense subsets of A to $\mathcal{T}$.