Commutative Algebra
Algèbre commutative
(Org: **Susan Cooper** (Manitoba), **Sara Faridi** (Dalhousie) and/et **Adam Van Tuyl** (McMaster))

ALI ALILOOEE, Bradley University
j-MULTIPLICITY OF EDGE IDEALS

The concept of j-multiplicity first was introduced by R. Achilles and M. Manaresi to generalize the Hilbert-Samuel multiplicity for ideals which are not m-primary. More precisely, let R be a Noetherian local ring with maximal ideal m and Krull dimension n and I be an ideal in R. We define the j-multiplicity $j(I)$ is defined as follow when $\ell(I) = \dim R$ (note that $\ell(I)$ is the analytic spread of I) and zero otherwise

$$j(I) = (n-1)! \lim_{k\to\infty} \frac{\lambda_R(\Gamma_m(I^k/I^{k+1}))}{k^{n-1}}.$$

Here Γ_m denotes the zeroth local cohomology with respect to the ideal m of R and λ denotes the length of Γ_m. In this talk first we will briefly survey some properties of j-multiplicity for monomial ideals and then we will see some results of j-multiplicity of the edge ideal of a graph.

LAURA BALLARD, Syracuse University
Properties of the Toric Ring of a Chordal Bipartite Family of Graphs

This project concerns the classification and study of a group of Koszul algebras coming from the toric ideals of a chordal bipartite infinite family of graphs (alternately, these rings may be interpreted as coming from determinants of subsets of certain matrices). I have determined a system of parameters for this family of rings, have developed a proof for the regularity of the rings and explicitly determined the Hilbert series for the Artinian reduction of these modulo a linear system of parameters, yielding, in particular, the multiplicity of the original rings. I will also report on partial progress on the nonadditive derived homological algebra of this family of algebras.

ASHWINI BHAT, University of Oklahoma
Generalized Borel ideals

In 2013, Francisco, Mermin, and Schweig introduced Q-Borel ideals, a generalization of Borel ideals defined by relations in a poset. We extend their work and describe some homological and combinatorial properties of these ideals.

RACHEL DIETHORN, Syracuse University
Koszul homology of quotients by edge ideals

We show that the Koszul homology algebra of a quotient by the edge ideal of a forest is generated by the lowest linear strand. This provides an answer, for such rings, to a question of Avramov about the Koszul homology algebra of Koszul algebras. We also recover a result of Roth and Van Tuyl on the graded Betti numbers of quotients of edge ideals of trees.

NEIL EPSTEIN, George Mason University
The space of graded Noether normalizations of a graded algebra

Let $S = k[x_1, \dots, x_n]$, k a field, and let J be a homogeneous ideal of S, with $d = \dim S/J$ and $c := n - d =$ height J. It is known that a sequence of linear forms $y_1, \dots, y_d \in \mathrm{Span}_k\{x_1, \dots, x_n\}$ induces a Noether normalization $k[y_1, \dots, y_d] \to S/J$ of S/J if and only if any basis of the k-vector space H they generate do so, and that this in turn is determined by the

c-dimensional dual space $H^\vee = \{v \in k^n : f(v) = 0 \quad \forall f \in H\}$. Thus viewing graded Noether normalizations as a subset of the Grassmannian $\mathbb{G}(c, n)$, let V be the complement of this set in the Grassmannian. Then V is a closed subvariety of $\mathbb{G}(c, n)$. Its defining ideal in the homogeneous coordinate ring D of $\mathbb{G}(c, n)$ (itself a well-behaved, though generally not regular, ring) has height 1, but we show that it otherwise reflects many of the properties of minimal primary decompositions of J itself. Indeed, we determine a well-behaved map that carries ideals of S to (not necessarily radical) ideals of D, which, especially when $k = \mathbb{C}$, is particularly well-behaved on homogeneous ideals of height $\geq c$ that are saturated with respect to S_+, and whose vanishing set is precisely the non-Noether normalization variety above. In particular, when $c = 1$, then $D \cong S$, and our map carries such an ideal J to an isomorphic copy of itself. This work is joint with Rebecca Goldin.

ZACH FLORES, Colorado State University
Macaulay Duals of Generic Hyperplane Arrangments

Given a generic hyperplane arrangement $\mathcal{A}$ with with defining polynomial $f_{\mathcal{A}}$, we ask when the Macaulay dual, $f_{\mathcal{A}}^{\perp}$ is a complete intersection. We discuss a lower bound on the initial degree of $f_{\mathcal{A}}^{\perp}$ that answers this for $|\mathcal{A}|$ sufficiently small, as well as discussing examples for when this lower bound can give no information.

CHRIS FRANCISCO, Oklahoma State University
Hilbert functions with a unique set of graded Betti numbers

Let $S = k[x_1, \ldots, x_n]$ be a polynomial ring over a field k. Given the graded Betti numbers for a module S/I, one can compute the Hilbert function of S/I. In the other direction, the Hilbert function imposes constraints on the graded Betti numbers (e.g., the lex ideal provides upper bounds), but in general there are many possible sets of graded Betti numbers for a given Hilbert function. We ask when the Hilbert function uniquely determines the graded Betti numbers for a module with that Hilbert function, giving some infinite families of examples of this behavior based on work of Evans and Richert.

FEDERICO GALETTO, Cleveland State University
Tangent schemes of determinantal varieties

Determinantal varieties are defined by the vanishing of all minors of a given size in a matrix of indeterminates. Their geometric and algebraic properties are generally well understood; in particular, we know how to compute the Betti numbers of their defining ideals. This talk will present work in progress on tangent schemes of determinantal varieties showing how, in some cases, we can similarly understand their geometric properties and also compute their Betti numbers.

FRANCESCA GANDINI, Kalamazoo College
Noether's degree bound in the exterior algebra

When we consider the action of a finite group on a polynomial ring, a polynomial unchanged by the action is called an invariant polynomial. A famous result of Noether states that in characteristic zero the maximal degree of a minimal invariant polynomial is bounded above by the order of the group. Our work establishes that the same bound holds for invariant skew polynomials in the exterior algebra. Our approach to the problem relies on a theorem of Derksen that connects invariant theory to the study of ideals of subspace arrangements. We adapt his proof over the polynomial ring to the exterior algebra, reducing the question to establishing a bound on the Castelnuovo-Mumford regularity of intersections of linear ideals in the exterior algebra. We prove the required regularity bound using tools from representation theory. In particular, the proof relies on the existence of a functor on the category of polynomial functors that translates resolutions of ideals of subspace arrangements over the polynomial ring to resolutions of ideals of subspace arrangements over the exterior algebra.

ELENA GUARDO, Università di Catania
Hilbert functions of schemes of double and reduced points

Given a valid Hilbert function H of a zero-dimensional scheme in $\mathbb{P}^2$, we show how to construct a set of fat points $Z \subset \mathbb{P}^2$ of double and reduced points such that H_Z, the Hilbert function of Z, is the same as H. In other words, we show that any valid Hilbert function H of a zero-dimensional scheme is the Hilbert function of some set of double and reduced points.

GRAHAM KEIPER, McMaster University
Decompositions of Toric Ideals of Finite Simple Graphs

I will discuss recent joint work which allows us to construct new finite simple graphs from two known ones in a specified way such that the corresponding toric ideals split. This construction more generally behaves well with respect to generators of the toric ideals of the graphs used in the construction. In some cases the technique allows us to recover the graded betti numbers of the resulting graph given that this information is known for the graphs used to construct it. Finally I hope to discuss more general results about the independence of generators of toric ideals.

CLAUDIA MILLER, Syracuse University
Resolutions for compressed Artinian algebras

We construct free resolutions of compressed Artinian graded algebra quotients of polynomial rings and give a method to reduce them to a minimal resolutions. Our result generalizes results of El Khoury and Kustin for Gorenstein algebras of even socle degree with a different proof.

This is joint work with Hamid Rahmati.

MIGUEL PACZKA, Instituto Politécnico Nacional
The Regularity of Some Families of Circulant Graphs

In this talk I will present explicit formulas for the Castelnuovo-Mumford regularity of the edge ideals of two families of circulant graphs, which includes all cubic circulant graphs. A feature of this approach is to combine bounds on the regularity, the projective dimension, and the reduced Euler characteristic to derive an exact value for the regularity.

JENNA RAJCHOT, University of Saskatchewan
Geometric vertex decomposition and liaison

Geometric vertex decomposition and liaison are two frameworks that are useful for studying classes of ideals in polynomial rings. These approaches were historically used by two distinct communities of mathematicians.

In this talk, I will connect these two approaches. In particular, I will show that each geometrically vertex decomposable ideal is linked by a sequence of ascending elementary G-biliaisons of height 1 to an ideal of indeterminates and, conversely, that each elementary G-biliaison of a certain type gives rise to a geometric vertex decomposition. As a consequence, I will show that several well-known families of ideals are glicci.

This is joint work with Patricia Klein.

BEN RICHERT, California Polytechnic State University
Computing Artinian Gorenstein k-algebras with the weak Lefschetz property and largest graded Betti numbers

Let k be a field and recall that an SI-sequence is a Hilbert function of a cyclic k-module whose first half is a differentiable $\mathcal{O}$-sequence. Harima demonstrated that the collection of SI-sequences characterizes the Artinian Gorenstien k-algebras with the weak Lefschetz property. Migliore and Nagel showed that fixing an SI-sequence $\mathcal{H}$, the collection of Betti diagrams for Artinian Gorenstein k-algebras with the weak Lefschetz property and Hilbert function $\mathcal{H}$ has a unique largest element. Establishing an upper bound for the Betti diagrams in question turns out to be the easier step – showing the bound is sharp requires a fairly difficult construction utilizing 'generalized stick figures.' We give a new proof that the bound is sharp by way of monomial ideals and a doubly iterative procedure, making the description more compact and the overall proof shorter and more naive (in the sense that it relies mostly on double induction). Admittedly, our machinery obfuscates the geometric intuition and insight inherent in Migliore and Nagel's pioneering work; we gain economy as well as computability since, being monomial until the last possible moment, it is easy to actually compute the ideals in question on the computer algebra system Macaulay 2 even for 'large' $\mathcal{H}$.

MAYADA SHAHADA, Dalhousie University
Simplicial Homology Splitting

If $S = k[x_1, \dots, x_n]$ is a polynomial ring over a field k and I is a graded ideal of S with minimal free resolution

$$0 \to \oplus_{j\in\mathbb{N}} S(-j)^{\beta_{p,j}} \to \oplus_{j\in\mathbb{N}} S(-j)^{\beta_{p-1,j}} \to \cdots \to \oplus_{j\in\mathbb{N}} S(-j)^{\beta_{1,j}} \to S,$$

then for each i and j, the rank $\beta_{i,j}(S/I)$ of the free S-modules appearing above are called the **graded Betti numbers** of the S-module S/I.

It is known that, in the class of monomial ideals, betti numbers can be interpreted as the homology of objects in discrete topology like simplicial complexes, order complexes of lattices, and more. In this talk, we will consider the question of when (or if) can we **split** up non-vanishing homology in these objects. In particular, if a simplicial complex Δ has non-vanishing homology in some dimension, then can the corresponding homological cycle induce homological cycles of smaller dimension? We will also see how splitting simplicial homology idea is related to the **sub-additivity** property of syzygies and use it to prove the sub-additivity property for some classes of monomial ideals.

SERGIO DA SILVA, University of Manitoba
Frobenius Splittings and the Desingularization of Hypersurfaces in Positive Characteristic

Frobenius splittings are useful tools for various questions in commutative algebra and algebraic geometry. For example, the Frobenius map for rings can be used to show that a given affine variety is reduced, and its extension to schemes appears in results involving Schubert varieties. I will show that Frobenius splittings can also be used to address the problem of resolving singularities in positive characteristic. In its simplest form, a resolution of singularities is a birational map from a smooth algebraic variety to a singular one. Desingularization in positive characteristic has remained a difficult problem, mostly because characteristic zero techniques fail in this setting. Working in the affine hypersurface case, I will show why curves and surfaces that define Frobenius splittings can be desingularized without alteration to the characteristic zero algorithm.

SANDRA SPIROFF, University of Mississippi
Ladder determinantal rings over normal domains

Ladder determinantal rings are generalizations of determinantal rings. The divisor class group and set of isomorphism classes of semi-dualizing modules for determinantal rings is known. We explicitly describe the divisor class group and semidualizing modules for ladder determinantal rings. Moreover, we work with coefficients in an arbitrary normal domain and arbitrary ladders, not necessarily connected, and all sizes of minors. This is joint work with Tony Se and Sean Sather-Wagstaff.