RAM MURTY, Queen's University
THE PALEY GRAPH CONJECTURE AND DIOPHANTINE TUPLES

Let n be a fixed natural number. An m-tuple $(a_1, ..., a_m)$ is said to be a Diophantine m-tuple with property $D(n)$ if $a_i a_j + n$ is a perfect square for i, j distinct and less than or equal to m. It is conjectured that the number of such tuples is bounded by an absolute constant. We will relate this question to the Paley graph conjecture which predicts the following. Let $\epsilon > 0$ be a real number, $S, T \subseteq \mathbb{F}_p$ for an odd prime p with $|S|, |T| > p^\epsilon$, and χ any nontrivial multiplicative character modulo p. Then, there is some number $\delta = \delta(\epsilon)$ for which the inequality

$$\left| \sum_{a \in S, b \in T} \chi(a+b) \right| \leq p^{-\delta} |S||T|$$

holds for primes larger than some constant $C(\epsilon)$. We show the Paley graph conjecture implies that the number of Diophantine m-tuples with property $D(n)$ is $O((\log n)^c)$ for any $c > 0$. This is joint work with Ahmet Güloğlu.