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Connections and moduli of bundles.

This talk concerns the symplectic geometric and differential geometric aspects of the moduli space of connections on a compact Riemann surface X . Fix a theta characteristic $K_X^{1/2}$ on X ; it defines a theta divisor on the moduli space \mathcal{M} of stable vector bundles on X of rank r degree zero. Given a vector bundle $E \in \mathcal{M}$ lying outside the theta divisor, we construct a natural holomorphic connection on E that depends holomorphically on E . Using this holomorphic connection, there is a canonical holomorphic isomorphism between:

1. the moduli space \mathcal{C} of pairs (E, D) , where $E \in \mathcal{M}$ and D is a holomorphic connection on E , and
2. the space $\text{Conn}(\Theta)$ given by the sheaf of holomorphic connections on the line bundle on \mathcal{M} associated to the theta divisor.

The above isomorphism between \mathcal{C} and $\text{Conn}(\Theta)$ is symplectic structure preserving, and it moves holomorphically as X runs over a holomorphic family of Riemann surfaces. (joint with I. Biswas)