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Ascent sliceness

A virtual link is an equivalence class of embeddings $\bigsqcup S^1 \hookrightarrow \Sigma_g \times I$, up to self-diffeomorphism of $\Sigma_g \times I$ and certain handle (de)stabilisations of Σ_g.

A cobordism between classical knots is a surface, properly embedded in $S^3 \times I$, which cobounds the knots. A cobordism between virtual knots $K : S^1 \hookrightarrow \Sigma_g \times I$ and $K' : S^1 \hookrightarrow \Sigma_{g'} \times I$ is a pair (S, M), for M a compact oriented 3-manifold with $\partial M = \Sigma_g \sqcup \Sigma_{g'}$, S an oriented surface properly embedded in $M \times I$ with $\partial S = K \sqcup K'$. If the genus of S is zero then (S, M) is a concordance. We may therefore ask new questions about the complexity of the 3-manifolds appearing in cobordisms between K and K'.

We outline one such question regarding the 3-manifolds appearing in concordances between virtual knots and the unknot. Roughly, given a virtual knot $K \hookrightarrow \Sigma_g$ and concordance (S, M) from K to the unknot, place a Morse function on M; the level sets are surfaces Σ_l. If there exists a level set Σ_l such that $l > g$ then the concordance is *ascent*. Does there exist a slice virtual knot such that every concordance between it and the unknot is ascent?