Recent Trends in Nonlinear Partial Differential Equations and Related Problems
Tendances récentes dans le domaine des équations aux dérivées partielles non linéaires et problèmes connexes
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YAKINE BAHRI, UVic
Transverse stability of line soliton for wave guide Schrödinger equations.

The transverse stability has been studied for 2D NLS equations in the spatial cylinder $\mathbb{R}\times\mathbb{T}$. It consists of studying the stability of the standing waves for 1D NLS under the 2D NLS flow.

In this talk, we consider the wave-guide Schrödinger equations

$$i\partial_t\psi + \partial_{xx}\psi - |D_y|\psi + |\psi|^{p-1}\psi = 0, \qquad \text{in } \mathbb{R}\times\mathbb{R}\times\mathbb{T},$$

where $1 < p < 5$, $|D_y| := \sqrt{-\partial_{yy}}$ and $\mathbb{T} = \mathbb{R}/2\pi\mathbb{Z}$. This equation was introduced by Xu who showed a modified scattering result.

We will show that the transverse stability depends on the frequencies ω. We will classify the result with respect to a critical frequency ω_p i.e. we will discuss the cases $0 < \omega < \omega_p$, $\omega > \omega_p$ and $\omega = \omega_p$.

This is a joint work with Hiroaki Kikuchi and Slim Ibrahim.

JUNHO CHOI, UNIST
On an Interior Layer for the Burgers Equations in $\mathbb{R}$

As a simplified model derived from the Navier-Stokes equations, we consider the viscous Burgers equations in $\mathbb{R}$,

$$\begin{aligned}
&u_t^\epsilon - \epsilon u_{xx} + \frac{(u^\epsilon)^2}{2} = f(x,t), \quad x\in\mathbb{R}, \quad t\geq 0\\
&u^\epsilon(x,0) = u_0(x), \quad x\in\mathbb{R},\\
&u^\epsilon \to g \ as\ x\to-\infty,\ u^\epsilon \to h\ as\ x\to\infty\ and\ g>0>h, \quad \forall t\geq 0.
\end{aligned} \tag{1}$$

We investigate the singular behaviors of their solutions u^ϵ as the viscosity parameter ϵ gets smaller. Indeed, when ϵ gets smaller, u_x^ϵ has viscous shocks whose slopes are proportial to $1/\epsilon$. So controlling the sharp layer is one of the most important parts in this research.

The idea is constructing the asymptotic expansions in the order of the ϵ and validating the convergence of the expansions to the solutions u^ϵ as $\epsilon\to 0$ in $L^2(0,T;H^1(\mathbb{R})))$ space. In this article, we consider the case where a single viscous shock occurs, i.e. interior layers, and we fully analyse the convergence at any order of ϵ using the so-called interior layer correctors

YANXIA DENG, University of Victoria
Global Existence and Singularity of the N-body Problem.

Unstable dispersive Hamiltonian evolution equations, such as semi-linear Klein-Gordon and Schrodinger equations, exhibit "soliton"-like solutions. Amongst those one singles out the ground state, which has the lowest energy of all solitons. When the solutions are restricted to have energies at most slightly above that of the ground state, one obtains a trichotomy in forward time for this regime of energies. Recently, we found that this idea could be used to characterize solutions in the N-body problem. The N-body problem is an initial-value problem for ordinary differential equations. The appropriate candidates of the ground states in the N-body problem are the relative equilibria, which are solutions whose configuration remains an isometry of the initial configuration. The characterization of solutions for the two-body problem in this new perspective is completely solved, and it resembles the results in PDE nicely. For $N > 2$, the question becomes quite complicated, and I will give some partial results.

STEPHEN GUSTAFSON, University of British Columbia
Chiral magnetic skyrmion solutions of 2D Landau-Lifshitz equations: existence.

Landau-Lifshitz equations are the basic dynamical equations in a micromagnetic description of a ferromagnet. They are naturally viewed as geometric evolution PDE of dispersive ("Schrödinger map") or mixed dispersive-diffusive type, which scale critically with respect to the physical energy in two dimensions. We describe results on existence and properties of important topological soliton solutions known as "chiral magnetic skyrmions". Joint work with Li Wang.

SLIM IBRAHIM, University of Victoria
Relativistic Vlasov-Maxwell for strongly magnetized Plasmas

An important challenge in plasma physics is to determine whether ionized gases can be confined by strong magnetic fields. After modelling, this question leads to a penalized version of the Relativistic Vlasov Maxwell system, marked by the role of a singular term coming from the inverse of a cyclotron frequency. In this talk, I will go over the well-posedness of the corresponding Cauchy problem. This is a joint work with C. Cheverry.

MASAHIRO IKEDA, RIKEN AIP Center
Well-posedness of fourth-order Schrödinger equation with derivative nonlinearities

In this talk, we study well-posedness of the Cauchy problem to the forth-order nonlinear Schrödinger equations with $\gamma \in \{1, 2, 3\}$-times derivative nonlinearities in Sobolev space $H^s(\mathbb{R})$:

$$\begin{cases} i\partial_t u + \partial_x^4 u = G\left((\partial_x^k u)_{k\leq\gamma}, (\partial_x^k \bar{u})_{k\leq\gamma}\right), & (t,x) \in I \times \mathbb{R}, \\ \quad u|_{t=0} = u_0 \in H^s(\mathbb{R}), \end{cases} \tag{2}$$

where $u : I \times \mathbb{R} \to \mathbb{C}$ is an unknown function, $I := [-T, T]$ denotes the existence time interval of the function u, $u_0 \in H^s(\mathbb{R})$ is a prescribed function, and for $s \in \mathbb{R}$, $H^s(\mathbb{R})$ denotes $L^2(\mathbb{R})$-based Sobolev space. For $m \in \mathbb{N}$ with $m \geq 3$, we mainly consider the m-th order nonlinearity G of the form

$$G(z) = G(z_1, \cdots, z_{2(\gamma+1)}) := \sum_{|\alpha|=m} C_\alpha z^\alpha,$$

where $C_\alpha \in \mathbb{C}$ with $\alpha \in (\mathbb{N} \cup \{0\})^{2(\gamma+1)}$ are constants. The purpose of this talk is to improve the previous results obtained by several Mathematicians, that is, to treat more general nonlinearity and to prove local well-posedness of the problem in lower order Sobolev space $H^s(\mathbb{R})$. Our proof of the well-posedness result is based on the contraction argument on a suitable function space, via the Strichartz estimates, Kato-type smoothing estimates, Kenig-Ruiz estimates, Maximal function estimates, a linear estimate for inhomogeneous term, the bilinear Strichartz type estimate and the Littlewood-Paley theory.

TAKAHISA INUI, Osaka University
Strichartz estimates for the damped wave equation and its application to the nonlinear problem

In this talk, we consider the damped wave equation (DW). The L^p-L^q type estimate was firstly obtained by Matsumura in 1976. After his work, many researchers have obtained such type estimates. However, there are less results for the space-time estimates, which are so called Strichartz estimates. Recently, Watanabe (2017) proved the Strichartz estimates for DW in the low space-dimensional case. In this talk, I show the Strichartz estimates in the higher dimensional case. Moreover, we consider the energy critical nonlinear damped wave equation (NLDW). Precisely, we discuss the local well-posedness, the decay property, and the finite time blow-up of the solutions to NLDW.

ISAO KATO, Department of Mathematics, Kyoto University
Local well-posedness for the Cauchy problem of the Zakharov type system

In this talk, we consider the Cauchy problem of the Zakharov type system. The system has no dispersion in some direction in the usual Zakharov system, so we call it the degenerated Zakharov system. The linear part of the degenerated Zakharov system is more complicated than that of the Zakharov system, so it is difficult to apply directly the local well-posedness result by Ginibre-Tsutsumi-Velo(1997). There are few well-posedness results for this system. The latest result is given by Barros-Linares(2015) for the three dimensional case, and they applied the linear estimate such as the maximal function estimate and the Strichartz estimate. To obtain the local well-posedness result with lower regularity initial data, we apply the Fourier restriction norm method. In this method, the norm of the function space is reflected in the linear part of the equation. Thus, the method is a very powerful tool to recover the derivative loss and we can obtain well-posedness with low regularity initial data. We treat the non-resonant part and the resonant part more carefully than in the case of the Zakharov system because of the lack of dispersion, then we obtain the system is locally well-posed in some anisotropic Sobolev space with low regularity.

HIROAKI KIKUCHI, Tsuda University
Minimization problem associated with ground states to combined power-type nonlinear Schrödinger equations

In this talk, we consider a minimization problem associated to ground state solutions to the following scalar field equation

$$-\Delta u + \omega u - |u|^{p-1}u - |u|^4 u = 0 \qquad \text{in } \mathbb{R}^3, \tag{3}$$

where $\omega > 0$ and $1 < p < 5$.

we shall show that the minimization problem has no minimizer when $\omega \gg 1$ and $1 < p \leq 3$. In addition, we determine an explicit frequency threshold that classifies the existence and non-existence.

To prove our results, we employ a resolvent expansion as in Coles and Gustafson (preprint). The resolvent expansion has a singularity because of a resonance in three space dimensions. The frequency threshold appears thanks to the presence of this singularity.

This talk is based on a joint work with T. Akahori(Shizuoka), S. Ibrahim (Victoria) and H. Nawa (Meiji)

NOBU KISHIMOTO, Kyoto University
Ill-posedness of the periodic nonlinear Schrödinger equation with third-order dispersion and Raman scattering term

We consider the nonlinear Schrödinger equation with third-order dispersion and derivative nonlinearity on the one-dimensional torus:

$$i\partial_t u + \partial_x^2 u + i\partial_x^3 u = c_1|u|^2 u + ic_2\partial_x(|u|^2 u) + i\gamma\partial_x(|u|^2)u, \qquad t \in \mathbb{R}, \quad x \in \mathbb{R}/2\pi\mathbb{Z},$$

where c_1, c_2 are real constants and γ is a complex constant. This equation is regarded as a mathematical model for the photonic crystal fiber oscillator, and the last term, with the coefficient γ having non-zero imaginary part, is related to the intrapulse Raman scattering effect, which is not negligible for ultrashort optical pulses. Without the Raman scattering term (i.e., $\mathrm{Im}\gamma = 0$), or for the non-periodic problem with any complex γ, the associated Cauchy problem is known to be locally well-posed in Sobolev spaces. We show that in the periodic setting the Raman scattering term causes ill-posedness (more precisely, non-existence of local-in-time solutions) of the Cauchy problem in Sobolev and Gevrey spaces. This talk is based on a joint work with Yoshio Tsutsumi (Kyoto University).

SHENGYI SHEN, University of Victoria
Analysis about a Two fluid mode and its comparison with MHD system

We study a two fluid system which describes the motion of two charged particles in a strict neutral incompressible plasma. We study the well-posdness of the system in both space dimensions two and three. Regardless of the size of the initial data, we prove the global well-posedness of the Cauchy problem when the space dimension is two. In space dimension three, we construct global weak-solutions, and we prove the local well-posedness of Kato-type solutions. These solutions turn out to be global when the initial data are sufficiently small. We also study the stability of the solution around zero given that the initial data is small and has sufficient regularity. It turns out that our system is a system of regularity-loss and the L^2 norm of lower derivatives of the solution decays. At last, this two fluid system can derive the classic MHD at least formally. Arsenio, Ibrahim and Masmoudi (2015) proved that the two fluid system converges to MHD under some constrain. We showed numerically that the two fluid system converges to MHD with no such constrain.

IKKEI SHIMIZU, Kyoto University
Remarks on local theory for Schrödinger maps near harmonic maps

In this talk, we consider the Schrödinger map equation (Landau–Lifshitz equation). In the study of the equation, one of the major method is to reduce it to some nonlinear Schrödinger equation, which is called the modified Schrödinger map equation. This talk will focus on the case of equivariant solutions under the Coulomb gauge condition. We show that the derivation of modified equation can be justified even from rough solutions. This result leads to an improvement on the uniqueness of solutions near harmonic maps.

KEISUKE TAKASAO, Department of Mathematics, Kyoto University
Global existence of weak solution for volume preserving mean curvature flow via phase field method

In this talk, we consider the phase field method for the volume preserving mean curvature flow. Given an C^1 hypersurface, we prove the global existence of the weak solution for the volume preserving mean curvature flow via the reaction diffusion equation with a non-local term. In particular, we show the L^2-boundedness of the mean curvature which is the key estimate to prove the main theorem. In addition, we prove the monotonicity formula for the reaction diffusion equation.

TAI-PENG TSAI, University of British Columbia
Existence, uniqueness, and regularity results for elliptic equations with drift terms in critical weak spaces

We consider Dirichlet problems for linear elliptic equations of second order in divergence form on a bounded or exterior smooth domain Ω in $\mathbb{R}^n$, $n \geq 3$, with drifts $\mathbf{b}$ in the critical weak L^n-space $L^{n,\infty}(\Omega;\mathbb{R}^n)$, and $\operatorname{div} \mathbf{b} \geq 0$ in $L^{n/2,\infty}(\Omega)$. We first establish existence and uniqueness of weak solutions in $W^{1,p}(\Omega)$ or $D^{1,p}(\Omega)$ for any p with $n' = n/(n-1) < p < n$. By duality, a similar result also holds for the dual problem. Next, we prove $W^{1,n+\epsilon}$ or $W^{2,n/2+\delta}$-regularity of weak solutions of the dual problem for some $\epsilon, \delta > 0$ when the domain Ω is bounded. By duality, these results enable us to obtain a quite general uniqueness result as well as an existence result for weak solutions belonging to $\bigcap_{p<n'} W^{1,p}(\Omega)$. Finally, we prove a uniqueness result for exterior problems, which implies in particular that (very weak) solutions are unique in both $L^{n/(n-2),\infty}(\Omega)$ and $L^{n,\infty}(\Omega)$. This is a joint work with Hyunseok Kim.

LI WANG, University of British Columbia
Chiral magnetic skyrmion solutions of 2D Landau-Lifshitz equations: stability.

Landau-Lifshitz equations are the basic dynamical equations in a micromagnetic description of a ferromagnet. They are naturally viewed as geometric evolution PDE of dispersive ("Schrödinger map") or mixed dispersive-diffusive type, which scale critically with respect to the physical energy in two dimensions. We describe results on the stability of important topological soliton solutions known as "chiral magnetic skyrmions". Joint work with Stephen Gustafson.

XINWEI YU, University of Alberta
A New Type of Regularity Criterion for the 3D Navier-Stokes Equations

We present a new kind of regularity criterion for the global well-posedness problem of the three dimensional Navier-Stokes equations in the whole space. The main novelty of this new criterion is that it involves the shape of the magnitude of the velocity. More specifically, we prove that if for every fixed time in $(0, T)$, the region of high velocity, appropriately defined with a parameter q, shrinks fast enough as $q \nearrow \infty$, then the solution stays regular beyond T. We further argue that reasonable flows satisfy our criterion, and singularity in Navier-Stokes is highly unlikely.

This is joint work with Prof. Chuong V. Tran of the University of St. Andrews, United Kingdom.