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On an Interior Layer for the Burgers Equations in $\mathbb{R}$

As a simplified model derived from the Navier-Stokes equations, we consider the viscous Burgers equations in $\mathbb{R}$,

$$\begin{aligned} &u^\epsilon_t - \epsilon u_{xx} + \frac{(u^\epsilon)^2}{2} = f(x,t), \quad x \in \mathbb{R}, \quad t \geq 0 \\ &u^\epsilon(x,0) = u_0(x), \quad x \in \mathbb{R}, \\ &u^\epsilon \to g \ as \ x \to -\infty, \ u^\epsilon \to h \ as \ x \to \infty \ and \ g > 0 > h, \quad \forall t \geq 0. \end{aligned} \tag{1}$$

We investigate the singular behaviors of their solutions u^ϵ as the viscosity parameter ϵ gets smaller. Indeed, when ϵ gets smaller, u^ϵ_x has viscous shocks whose slopes are proportional to $1/\epsilon$. So controlling the sharp layer is one of the most important parts in this research.

The idea is constructing the asymptotic expansions in the order of the ϵ and validating the convergence of the expansions to the solutions u^ϵ as $\epsilon \to 0$ in $L^2(0,T;H^1(\mathbb{R})))$ space. In this article, we consider the case where a single viscous shock occurs, i.e. interior layers, and we fully analyse the convergence at any order of ϵ using the so-called interior layer correctors