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Degenerate nonlinear diffusion equations with discontinuous nonlinearities

In this talk, I will consider Cauchy problems for degenerate diffusion equations of the form $\partial_t\rho - \Delta\varphi(\rho) = 0$, $\rho(0,\cdot) = \rho_0$ on smooth bounded domains of $\mathbb{R}^d$, where ρ_0 is a given probability measure. The increasing nonlinearity $\varphi : [0,+\infty) \to \mathbb{R}$ is supposed to have a discontinuity at some $s_0 \in (0,+\infty)$. Such models arise in mathematical biology describing so-called *self-organized criticality* phenomena. To show the well-posedness of such problems, for a large class of nonlinearities, we rely on its gradient flow formulation in the space of probability measures equipped with the so-called L^2-*Wasserstein distance*. We show that in general a 'critical zone', $\{\rho = s_0\}$ emerges, and the problem gives rise to a three-phase Stefan-type problem. We will show that a *pressure term* appears in the precise description of the solution, which is concentrated on the critical zone. By this observation, we can make a link to recent models on congested crowd dynamics. The talk is based on a joint work with Dohyun Kwon (UCLA).