Harmonic Analysis and its Connections to Geometric Measure Theory and Combinatorics
Analyse harmonique et liens avec la théorie des mesures géométriques et la combinatoire
(Org: **Kyle Hambrook** (San Jose State University) and/et **Itay Londner** (Tel Aviv University))

SIMON BORTZ, University of Washington
Two-Phase Free Boundary Problems for Harmonic Measure

In this talk we give a (two-phase) potential theoretic characterization of two-sided vanishing chord arc domains under minimal background hypotheses. This problem is most closely related to the work of Kenig and Toro, where they show a similar result under stronger topological assumptions. We are able to dispense with these topological assumptions by using powerful tools from classical geometric analysis combined with techniques coming from the theory of uniformly rectifiable sets (introduced by David and Semmes). This is joint work with Max Engelstein, Max Goering, Tatiana Toro and Zihui Zhao.

MARCIN BOWNIK, Univeristy of Oregon
Wavelets for non-expanding dilations

In this talk we we discuss the problem of existence and characterization of wavelets for non-expanding dilations. This turns out to be intimately connected with the geometry of numbers; more specifically, with the estimate on the number of lattice points of dilates of balls by the powers of a dilation $A \in GL_n(\mathbb{R})$. This connection is not visible for the well-studied class of expanding dilations since the desired lattice counting estimate holds automatically.

We show that the lattice counting estimate holds for all dilations A with $|\det A| \neq 1$ and for almost every lattice Γ with respect to invariant measure on the set of lattices. As a consequence, we deduce the existence of minimally supported frequency (MSF) wavelets associated with such dilations for almost every choice of a lattice. Likewise, we show that MSF wavelets exist for all lattices and and almost every choice of a dilation A with respect to the Haar measure on $GL_n(\mathbb{R})$.

This talk is based on a joint work with Jakob Lemvig.

MARC CARNOVALE, The Ohio State University
Abundance of 3APs in sparse subsets of integers, finite fields, and Euclidean spaces

Motivated by the study of patterns in the prime numbers, in 1936 Erdős and Turán conjectured that any dense subset of the natural numbers must contain arbitrarily long arithmetic progressions. In 1953, Roth proved the 3-term case of this conjecture (later resolved in full by Szemerédi in '75) by Fourier analytic means. In 2008, Laba and Pramanik showed that an analog of the pseudorandom half of Roth's approach could be made to work in the fractal setting. As the field of additive combinatorics has come into its own in the past two decades, the study of arithmetic progressions within various sets, and, centrally, Roth's and Szemerédi's theorems, have continued to grow in significance. In this talk we will focus on the tri-directional interplay between harmonic analysis, geometric measure theory, and discrete additive combinatorics through the lens of recent work of the speaker (partially joint with Steven Senger).

JACOB DENSON, University of British Columbia
Fractals Avoiding Fractal Configurations

What's the largest Hausdorff dimension a Euclidean set can have avoiding configurations? How large can a set of real numbers be avoiding arithmetic progressions, or subsets of the plane not containing vertices of equilateral triangles? Recent work gives general methods to find sets avoiding configurations specifiable by a smooth function, like the ones above. In this talk, we discuss extensions of these methods to 'fractally specified configurations'. Applying our method, for instance, gives high dimensional sets whose difference sets avoid rational translates of the Cantor set. We introduce the configuration discretization method standard in the field, and explain the idea behind our randomized configuration avoidance method.

ED GRANIRER, University of British Columbia
Some Geometric Properties of the Fourier Algebras.

Some Banach space geometric properties, such as the Schur the Dunford-Pettis and the Radon-Nikodym Properties for the above algebras are investigated.

RACHEL GREENFELD, Bar-Ilan University
Fuglede's spectral set conjecture

A set $\Omega \subset \mathbb{R}^d$ is called *spectral* if the space $L^2(\Omega)$ has an orthogonal basis of exponential functions. Back in the 1970's Fuglede conjectured that spectral sets could be characterized geometrically by their ability to tile the space by translations. Although since then the subject has been intensively studied, the precise connection between spectrality and tiling by translations is still a mystery. In the talk I will survey the subject and discuss some recent results, joint with Nir Lev, where we focus on the conjecture for convex polytopes.

KYLE HAMBROOK, San Jose State University
Construction of fractal measures with rapid Fourier decay

I will discuss some of my recent work on problems connected to the construction of measures with fractal support and whose Fourier transform decays rapidly at infinity.

MARINA ILIOPOULOU, UC Berkeley
Counting joints formed by lines and a k-plane

A joint formed by a set of lines in $\mathbb{F}^n$ (where $\mathbb{F}$ is a field) is a point in $\mathbb{F}^n$ through which at least n of the lines pass, with the property that their directions span $\mathbb{F}^n$. The problem of counting joints relative to the number of lines forming them was solved in 2009 by Quilodrán and independently by Kaplan, Sharir and Shustin, using the polynomial method. However, the problem of counting joints formed by higher dimensional planes is much more difficult. In this talk we will discuss some very small progress in this direction, in the case where each of the joints is formed by lines and a k-dimensional plane. This is joint work with A. Carbery.

ALEX IOSEVICH, University of Rochester
The Falconer distance conjecture in the plane

We shall describe the recent result due to Guth, Iosevich, Ou, and Wang, proving that if the Hausdorff dimension of a compact planar set is greater than five quarters, then the Lebesgue measure of the distance set is positive. Some related problems in geometric measure theory will be discussed as well.

JONGCHON KIM, University of British Columbia / PIMS
Elliptic difference equations with random coefficients

It is a classical result that Green's functions for uniformly elliptic partial differential equations in divergence form are controlled by the Green's function for the Laplace's equation. When the coefficients are random variables, better regularity results are available for averaged Green's functions both in continuous and discrete settings. In this talk, we consider the averaged Green's function for an elliptic difference operator on the lattice $\mathbb{Z}^d$ with a coefficient matrix that is an i.i.d. perturbation of the identity matrix. In this setting, Bourgain proved a decay estimate for the kernel of an averaged operator which governs the averaged solution. We present some ingredients of his proof and also an improved estimate. As a corollary, sharp decay estimates for higher order derivatives of the averaged Green's function are shown to be valid up to $d + 1$ derivatives. This is a joint work with Marius Lemm.

IZABELLA LABA, UBC
Maximal operators and decoupling for Lambda(p) Cantor measures

We construct a class of Cantor-type measures for which the associated maximal operators obey a Sobolev-L^p estimate. This complements an earlier result of Laba-Pramanik in that the bound we obtain is weaker, but applicable under less restrictive assumptions. In particular, we are able to include measures supported on self-similar sets and sets of arbitrarily low Hausdorff dimension. The proof is based on multiplier estimates obtained via decoupling.

CHUN-KIT LAI, San Francisco State University
Existence and exactness of exponential Riesz sequences and frames for fractal measures

We study the construction of exponential frames and Riesz sequences for fractal measures using the idea of frame towers and Riesz-sequence towers. We show that the exactness and overcompleteness of the constructed exponential frame or Riesz sequence is completely classified in terms of the cardinality at each level of the tower.

Furthermore, using a version of the solution of the Kadison-Singer problem, known as the R_ϵ-conjecture, we show that all these measures contain exponential Riesz sequences of infinite cardinality. When the measure is the middle-third Cantor measure, or more generally for self-similar measures with no-overlap condition, there are always exponential Riesz sequences of maximal possible Beurling dimension. This leads to another positive evidence that middle-third Cantor measures may exist a Fourier frame.

This is a joint work with D. Dutkay and S. Emami

ITAY LONDNER, UBC
Exponential frames and syndetic Riesz sequences

In my talk I will discuss the following problem: Given a subset of the circle of positive Lebesgue measure $S \subset \mathbb{T}$, does there exist a subset of the integers $\Lambda \subset \mathbb{Z}$ with bounded gaps between consecutive elements, and such that the exponential system $E(\Lambda) := \{e^{i\lambda t}\}_{\lambda \in \Lambda}$ is a Riesz sequence in $L^2(S)$.

The solution to this problem is based on an application of the recent solution to the Kadison-Singer problem. In some cases an explicit (i.e. non-probabilistic) construction is attainable, using Fourier quasicrystals.

Joint work with Marcin Bownik (University of Oregon).

NEIL LYALL, University of Georgia
Geometric Configurations in Sets of Positive Density

We shall describe some recent results on the existence of certain prescribed geometric configurations in a variety of settings.

SHAHAF NITZAN, Georgia Institute of Technology

MALABIKA PRAMANIK, University of British Columbia, Vancouver
On Salem sets avoiding solutions of linear equations

The talk will provide a survey of the literature on sets of large Fourier dimension that do not contain any nontrivial solution of certain linear equations. We will report on ongoing work in this direction with Yiyu Liang.

K. S. SENTHIL RAANI, IISER Berhampur, Odisha, India
Role of curvature measures in determining Fourier asymptotics

The geometry of a set plays a vital role in studying the Fourier asymptotics. Surveying a few results in this connection, in this talk, we look at some developments concerning the sets that admit curvature measures which were introduced by Herbert Federer in 1959.

JORIS ROOS, University of Wisconsin-Madison
Averages of the simplex Hilbert transform

The simplex Hilbert transform is a multilinear operator generalizing the classical Hilbert transform. It is a difficult open problem in harmonic analysis to decide whether this operator satisfies any L^p bounds. In this talk I will explain some joint work in progress with Polona Durcik, where we study a certain averaged version of the simplex Hilbert transform which is related to the simplex Hilbert transform in the same way as the bilinear Hilbert transform is related to the Calderón commutator. In particular, our bounds imply some of the known bounds for higher-order commutators.

STEVEN SENGER, Missouri State University
Chains and other multiple point configurations

Our primary motivation is the classical Erdős unit distance problem, which asks for how often a unit distance can occur in a large finite point set in the plane. We generalize this to consider chains, or $(k+1)$-tuples of points with the k distances between consecutive points prescribed. We also discuss related results with more distances prescribed between points, and related problems in geometric measure theory.

BETSY STOVALL, University of Wisconsin-Madison
Extremizability of Fourier restriction to the paraboloid

We will show that for almost all valid $L^p(\mathbb{R}^{d+1}) \to L^q(\mathbb{P}^d)$ restriction inequalities, there exist functions of $L^p(\mathbb{R}^{d+1})$ norm 1 whose extensions have $L^q(\mathbb{P}^d)$ norm equal to the operator norm. As time permits, we will also discuss related questions for other Fourier restriction/extension operators.

KRYSTAL TAYLOR, The Ohio State
Lipschitz Images of Fractal Sets and an Application to Pinned Distance Sets

Fractals are sets with intricate structure at infinitely many scales. One robust way to deal with such arbitrary objects is to decompose them into more usable components. Two powerful methods of decompositions include the Fourier transform and projection theorems.

In this talk, we use these tools to establish relationships between the dimension (Hausdorff or Fourier dimension) of a pair of thin sets and the interior, dimension, and measure of the images of the pair under families of Lipschitz maps. For instance, in a joint work with K. Hambrook, we consider lower bounds on the dimension of the product set XY, where X is a set of scalars and Y is a subset of Euclidean space. In a joint work with K. Simon, we determine the measure of $X + S^1$ when X is a set of Hausdorff dimension 1, as well as the interior of $X + S^1$ when X is a suitable Cartesian product of Cantor sets. We then use these result to study distance sets, and we give the first known result in the literature on the interior of pinned distance sets.

HONG WANG, MIT
A restriction estimate in $\mathbb{R}^3$

If f is a function supported on a truncated paraboloid, what can we say about Ef, the Fourier transform of f? Stein conjectured in the 1960s that for any $p > 3$, $\|Ef\|_{L^p(R^3)} \lesssim \|f\|_{L^\infty}$. We make a small progress toward this conjecture and show that $p > 3 + 3/13 \approx 3.23$. In the proof, we combine polynomial partitioning techniques introduced by Guth and the two ends argument introduced by Wolff and Tao.

THOMAS YANG, University of British Columbia
Small sets containing many patterns

We show that if a subset A of $\mathbb{R}$ contains an affine copy of all bounded decreasing sequences, then A must be somewhere dense. On the other hand, given a collection $\mathcal{C}$ of bounded decreasing sequences that decays faster than a fixed threshold sequence, there is a closed and nowhere dense A that contains an affine copy of every sequence in $\mathcal{C}$.

XU ROBERT YANG, University of Waterloo
Interpolation Sets

A Sidon set E is a set with the property that we can interpolate every bounded function on E by a Fourier transform of a measure. In this talk we will discuss the decomposition of Sidon sets into even more special sets. Pisier and Bourgain showed that Sidon sets are proportional quasi-independent. We will show Sidon sets can be proportional k-independent for all $k \geq 1$ for a torsion-free discrete abelian group. Beased on that we will show Sidon sets are proportional Sidon with arbitrarily small Sidon constants.

JOSH ZAHL, University of British Columbia
An improved bound on the Hausdorff dimension of Besicovitch sets in $\mathbb{R}^3$

A Besicovitch set is a compact set in $\mathbb{R}^d$ that contains a unit line segment pointing in every direction. The Kakeya conjecture asserts that every Besicovitch set in $\mathbb{R}^d$ must have Hausdorff dimension d. I will discuss a recent improvement on the Kakeya conjecture in three dimensions, which says that every Besicovitch set in $\mathbb{R}^3$ must have Hausdorff dimension at least $5/2 + \epsilon$, where ϵ is a small positive constant. This is joint work with Nets Katz.

RUIXIANG ZHANG, UW-Madison
The pointwise convergence problem for the free Schrödinger equation in high dimensions

Recently a problem proposed by Carleson on a.e. convergence for free Schrödinger solutions got a sharp answer up to the endpoint in all dimensions. We will talk about the new result in dimensions $n + 1$ for all $n > 2$ and ideas behind it (joint work with Xiumin Du).