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Existence of high rank regular polytopes for PSp(4,q)

This talk pertains to the following general question: given an infinite family $\mathcal{F}$ of finite simple groups, and an integer $r > 2$, determine the members of $\mathcal{F}$ that arise as symmetries of an abstract regular polytope of rank r. It is unlikely that a complete answer will be found for all $\mathcal{F}$ and all r, but the question helps to structure the search for new families of polytopes having symmetry groups that by some measure are well understood.

The question has been settled for all families $\mathcal{F}$ when $r = 3$, and for the alternating groups $\mathrm{Alt}(n)$ and the projective groups $\mathrm{PSL}(3, q)$ and $\mathrm{PSU}(3, q)$ for arbitrary rank r. In joint work with Leemans I showed that $\mathrm{PSL}(4, q)$ has polytopes of rank 4, and with Ferrara and Leemans that $\mathrm{PSp}(2m, q)$ has rank $2m + 1$ polytopes so long as $q = 2^e$ is even. In this talk I will focus on the groups $\mathrm{PSp}(4, q)$ for q odd, giving constructions of polytopes of ranks 4 and 5.