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A Bombieri-Vinogradov Theorem for Modular Forms

For any $(a, q) = 1$, assuming the generalised Riemann hypothesis for Dirichlet L-functions, we have a strong form of Dirichlet's theorem on arithmetic progressions:

$$\pi(x, a, q) = \frac{1}{\phi(q)}\mathrm{Li}(x) + O(x^{1/2}\log(qx)),$$

where $\pi(x, a, q)$ stands for the number of primes $p \leq x$ congruent to a modulo q, ϕ is Euler's totient function, and $\mathrm{Li}(x)$ is the logarithmic integral function. Nowadays, although the generalised Riemann hypothesis remains open, we still know that such an estimate is valid "on average" by the celebrated theorem of Bombieri and Vinogradov.

In this talk, we will consider a modular variant of both theorems that gives a count of Fourier coefficients of modular forms over arithmetic progressions. If time allows, we will also discuss some applications related to questions of Lehmer and Serre on the non-vanishing of Fourier coefficients of modular forms.