Asymptotic Analysis and Applications
Analyse asymptotique et applications

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JUNHO CHOI, UNIST

On Boundary Layers for the Burgers Equations in a Bounded Domain

As a simplified model derived from the Navier-Stokes equations, we consider the viscous Burgers equations in a bounded domain with two-point boundary conditions.

$$\begin{aligned} &u_t^\epsilon - \epsilon u_{xx} + \frac{(u^\epsilon)^2}{2} = f(x,t), \quad x \in (0,1), \quad t \geq 0 \\ &u^\epsilon(x,0) = u_0(x), \quad x \in (0,1), \\ &u^\epsilon(0,t) = g(t), \quad t \geq 0, \\ &u^\epsilon(1,t) = h(t), \quad t \geq 0. \end{aligned} \tag{1}$$

We investigate the singular behaviors of their solutions u^ϵ as the viscosity parameter ϵ gets smaller. Indeed, when ϵ gets smaller, u_x^ϵ has $1/\epsilon$ order slope. So controlling the sharp slopes is one of the most important parts in this research.

The idea is constructing the asymptotic expansions in the order of the ϵ and validating the convergence of the expansions to the solutions u^ϵ as $\epsilon \to 0$ in $L^2(0,T;H^1((0,1)))$ space. In this article, we consider the case where sharp slopes occur at the boundaries, i.e. boundary layers, and we fully analyse the convergence at any order of ϵ using the so-called boundary layer correctors as follows.

In the end, we also numerically verify the convergences.

HOWARD COHL, National Institute of Standards and Technology

Asymptotics of Fundamental Solutions for Helmholtz operators on Spaces of Constant Curvature

We compute closed-form expressions for oscillatory and damped spherically symmetric fundamental solutions of the Helmholtz equation in d-dimensional hyperbolic and hyperspherical geometry. We are using the R-radius hypersphere and R-radius hyperboloid model of hyperbolic geometry. These models represent Riemannian manifolds with positive constant and negative constant sectional curvature respectively. Flat-space limits with their corresponding asymptotic representations, are used to restrict proportionality constants for these fundamental solutions. In order to accomplish this, we summarize and derive new large degree asymptotics for associated Legendre and Ferrers functions of the first and second kind. Furthermore, we prove that our fundamental solutions on the hyperboloid are unique due to their decay at infinity. To derive Gegenbauer polynomial expansions of our fundamental solutions for Helmholtz operators on hyperspheres and hyperboloids, we derive a collection of infinite series addition theorems for Ferrers and associated Legendre functions which are generalizations and extensions of the addition theorem for Gegenbauer polynomials. Using these addition theorems, in geodesic polar coordinates for dimensions greater than or equal to three, we compute Gegenbauer polynomial expansions for these fundamental solutions, and azimuthal Fourier expansions in two-dimensions.

DAN DAI, City University of Hong Kong

Gap probability at the hard edge for random matrix ensembles with pole singularities in the potential

We study the Fredholm determinant of an integrable operator acting on the interval $(0,s)$ whose kernel is constructed out of the Ψ-function associated with a hierarchy of higher order analogues to the Painlevé III equation. This Fredholm determinant describes the critical behavior of the eigenvalue gap probability at the hard edge of unitary invariant random matrix ensembles perturbed by poles of order k in a certain scaling regime. Using the Riemann-Hilbert method, we obtain the large s asymptotics of the Fredholm determinant. Moreover, we derive a Painlevé type formula of the Fredholm determinant, which is expressed in terms of an explicit integral involving a solution to a coupled Painlevé III system.

This is a joint work with Shuai-Xia Xu and Lun Zhang.

MOURAD ISMAIL, University of Central Florida
The q-Normal Distribution

We point out some unusual properties of the weight function of the q-Hermite polynomials observed by P. Szablowski and X. Yang. We provide rigorous proofs and show that many other polynomials of the q-Askey Scheme share the same properties. We also indicate how this leads to general questions about zeros of complex orthogonal polynomials.

XIANG-SHENG WANG, University of Louisiana at Lafayette
Asymptotic analysis of difference equations

In this talk, we will present some preliminary results on the asymptotic analysis of Wilson polynomials and q-orthogonal polynomials via difference equations. The first result is based on an ongoing joint work with Prof. Yu-Tian Li and Prof. Roderick Wong. The second result is based on an ongoing joint work with Prof. Dan Dai and Prof. Mourad Ismail.

MICHAEL WARD, University of British Columbia
The Stability of Hotspot Patterns for a Continuum Model of Urban Crime and the Effect of Police Intervention

In a singularly perturbed limit, we analyze the existence and linear stability of steady-state localized hotspot solutions for the 1-D three-component reaction-diffusion (RD) system formulated and studied numerically in Jones et. al. [Math. Models. Meth. Appl. Sci., **20**, Suppl., (2010)], which models urban crime with police intervention. In our model, the field variables are the attractiveness field for burglary, the criminal density, and the police density, and it includes a scalar parameter that determines the strength of the police drift towards maxima of the attractiveness field. For a special choice of this parameter, we recover the "cops-on-the-dots" policing strategy of Jones et. al., where the police mimic the drift of the criminals towards maxima of the attractiveness field.

For this model we develop a spectral theory based on the analysis of of nonlocal eigenvalue problems to provide phase diagrams in parameter space characterizing the linear stability of hotspot patterns. In one particular parameter regime, the hotspot steady-states are shown to be unstable to asynchronous oscillatory instabilities in the hotspot amplitudes arising from a Hopf bifurcation. Within the context of our model, this provides a parameter range where the effect of a cops-on-the-dots policing strategy is to only displace crime temporally between neighboring spatial regions. In other parameter regimes, we show that new hotspots of criminal activity can be nucleated in low crime regions when the spatial extent of these quiescent regions exceeds a critical threshold.

Both the mathematical challenges in the linear stability analysis, and the qualitative interpretation of our results are highlighted.

RODERICK WONG, City University of Hong Kong
Asymptotics of the associated Pollaczek polynomials (Joint with Min-Jie Luo)

In this talk, we present the large-n behavior of the associated Pollaczek polynomials $P_n^\lambda(z; a, b, c)$. These polynomials involve four real parameters λ, a, b and c, in addition to the complex variable z. Asymptotic formulas are derived for these polynomials, when z lies in the complex plane bounded away from the interval of orthogonality $(-1, 1)$, as well as in the interior of the interval of orthogonality.

SHUAI-XIA XU, Sun Yat-sen Univesity
Gap probability in critical unitary random matrix ensembles and the coupled Painlevé II system

We study Fredholm determinants of the Painlevé II and Painlevé XXXIV kernels. In certain critical unitary random matrix ensembles, these determinants describe special gap probabilities of eigenvalues. We obtain Tracy-Widom formulas for the Fredholm determinants, which are explicitly given in terms of integrals involving a family of distinguished solutions to the coupled Painlevé II system in dimension four. Moreover, the large gap asymptotics for these Fredholm determinants are derived, where the constant terms are given explicitly in terms of the Riemann zeta-function. This talk is based on a joint work with Dan Dai.

RUIMING ZHANG, Northwest Agriculture and Forestry University
Asymptotics of Theta Functions

In this talk we present two asymptotic expansions of theta functions with respect to two different scalings, both of them have exponential remainders. We also apply the asymptotics to compute asymptotic behaviors of partial sums of elliptic hypergeometric series.