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On Boundary Layers for the Burgers Equations in a Bounded Domain

As a simplified model derived from the Navier-Stokes equations, we consider the viscous Burgers equations in a bounded domain with two-point boundary conditions.

$$\begin{aligned}
&u^\epsilon_t - \epsilon u_{xx} + \frac{(u^\epsilon)^2}{2} = f(x,t), \quad x \in (0,1), \quad t \geq 0\\
&u^\epsilon(x,0) = u_0(x), \quad x \in (0,1),\\
&u^\epsilon(0,t) = g(t), \quad t \geq 0,\\
&u^\epsilon(1,t) = h(t), \quad t \geq 0.
\end{aligned} \tag{1}$$

We investigate the singular behaviors of their solutions u^ϵ as the viscosity parameter ϵ gets smaller. Indeed, when ϵ gets smaller, u^ϵ_x has $1/\epsilon$ order slope. So controlling the sharp slopes is one of the most important parts in this research.

The idea is constructing the asymptotic expansions in the order of the ϵ and validating the convergence of the expansions to the solutions u^ϵ as $\epsilon \to 0$ in $L^2(0,T;H^1((0,1)))$ space. In this article, we consider the case where sharp slopes occur at the boundaries, i.e. boundary layers, and we fully analyse the convergence at any order of ϵ using the so-called boundary layer correctors as follows.

In the end, we also numerically verify the convergences.