STUDC Research Session
Session de recherche du Comité des étudiants
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ADELA GHERGA, The University of British Columbia
Implementing algorithms to compute elliptic curves over Q

Let $S = \{p_1, \ldots, p_v\}$ be a set of rational primes. A theorem of Bennett-Rechnitzer shows that the problem of computing all elliptic curves over $\mathbb{Q}$ having good reduction outside S and bounded conductor N reduces to solving a number of Thue-Mahler equations. These are Diophantine equations of the form

$$F(x, y) = u,$$

where

$$F(x, y) = c_0 x^n + c_1 x^{n-1} y + \cdots + c_{n-1} x y^{n-1} + c_n y^n$$

is a given binary form of degree at least 3 and u is an S-unit. To solve such equations, there exists a practical method of Tzanakis-de Weger using linear forms in p-adic logarithms and various reduction techniques. In this talk, we describe our implementation of this method and discuss the key steps used in our algorithm, as well as the implications to computing elliptic curves.

AVLEEN KAUR, University of Manitoba
The eigenvalue problem of the Uzawa pressure operator

Knowledge about the spectrum of the Uzawa pressure operator, defined by $S := \nabla \cdot \Delta^{-1} \nabla$, is important for solving and performing an error analysis of the Stokes problem. The infimum of the spectrum of the Uzawa pressure operator, denoted by $\lambda_{\min}(S)$, is significant, for instance, it gives information about the rate of convergence of numerical methods for the Stokes problem. The spectrum of the Uzawa pressure operator is still not known for the case of a square domain. This talk describes some results related to this problem. It depicts the efforts made for estimating $\lambda_{\min}(S)$ for a square domain. In 1996, M. Gaultier and M. Lezaun gave an upper bound equal to 0.2260 for $\lambda_{\min}(S)$, we have improved it to 0.20164. We conjecture that $\lambda_{\min}(S) = \frac{1}{2} - \frac{1}{\pi} \approx 0.18169$ for a square domain.

ALICE LACAZE-MASMONTEIL, Acadia University
The game of Ambush Cops and Robbers played on the products of graphs

The game of Ambush Cops and Robbers is a variation of the game of Cops and Robber played on graphs. This variation is played with two robbers that are given the power to ambush a cop and win by both moving to the same vertex as a cop on the same turn. Otherwise, rules of this variation follow that of the game of Cops and Robber. Much of the existing research focusses on characterizing graphs on which a single cop has an ambush-winning strategy. A natural direction is to investigate classes of graphs for which ambush-winning strategies for multiple cops exist. A particular set of graphs of interest are those generated by graph products such as the strong, Cartesian, and categorical products. In this talk, the ambush-copnumbers of the aforementioned products are discussed in terms of the ambush-copnumbers of their factors. Comparisons with the original game are made by discussing challenges posed by the robbers' power. This is joint work with Dr. Nancy Clarke.

SARAH NATAJ AND S. H. LUI,
Superlinear convergence of quasi-Newton methods based on assumptions about the initial point

Broyden's method is a quasi-Newton method which is used to solve a system of nonlinear equations. Almost all convergence theory in the literature assumes existence of a root and bounds on the nonlinear function and its derivative in some neighborhood

of the root. All these conditions cannot be checked in practice. The motivation of this talk is to derive a convergence theory for Broyden where all assumptions can be verified, and the existence of a root and its superlinear rate of convergence are consequences of the theory. The method of BFGS is a quasi-Newton method for unconstrained minimization. Also, all known convergence theory assume existence of a solution and bounds of the function in a neighborhood of the minimizer. The other part of this talk would be on convergence theory of BFGS method where all assumptions are verifiable and existence of a minimizer and also discussing the superlinear convergence of the iteration.

PIERRE-OLIVIER PARISÉ, Université Laval
The Impossibility of Polynomial Approximations in Some de Branges-Rovnyak Spaces

In 1993, D. Sarason wrote a book in which he summarized the results of a fruitful topic relating operator theory and complex analysis: the so-called de Branges-Rovnyak spaces $\mathcal{H}(b)$. These spaces are parametrized by a holomorphic function b bounded by 1 on the unit disk and are sub-Hilbert spaces of the Hardy space H^2.

In this talk, we introduce the de Branges-Rovnyak spaces. Our goal is to present a criterion on the function b which is necessary and sufficient for the set of polynomials to be dense in the space $\mathcal{H}(b)$. When b does not satisfy the criterion, most polynomials are not even in the space. In this situation, the theory of de Branges-Rovnyak spaces differs from the theory of the classical spaces of holomorphic functions in the unit disk such as the Hardy space, the Dirichlet space and the Bergman space. Indeed, the set of polynomials belongs to these classical spaces and forms a dense subset.

JOHN SASTRILLO, University of British Columbia
Analyzing the Nature of Citations and Their Automatic Classification

With the ever growing number of scientific publications, navigating a field's literature for unexplored possibilities becomes correspondingly more difficult. While citations are useful in connecting the progression of ideas over the years, a common limitation persists in the lack of context for why a publication was cited. Often times, the context needs to be inferred from the text. While various platforms have attempted to present the context of citations in different ways, they often fail to give a clear definition of the nature of the citation. Instead, these platforms present the sentence where the citation is found but the sentence again lacks context without reading the entire text. In this presentation, I will discuss the pros and cons of both Machine Learning and non-Machine Learning algorithms in finding the best sentence to describe the nature of a citation and classifying the citation based on the sentence found. Furthermore, I will discuss how different platforms have used citations to facilitate researchers' understanding of their fields. With clearer definitions for why publications are cited in conjunction with an automatic way to describe the nature of a citation, we can create effective metrics for understanding the impact of a publication, and develop novel platforms for visualizing, navigating and understanding the changes in a field. On a broader level, a deeper understanding of the progression of a field overtime can highlight new areas and challenges for future research.