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Saturated Free Algebras and Almost Indiscernible Theories

[With Anand Pillay, University of Notre Dame.]

We extend to uncountable languages the studies of the model theory of saturated free algebras in countable languages by [Baldwin, Shelah (1983)], and generalizations by [Pillay, Sklinos (2015)].

A theory T of cardinality τ is (μ, κ)-almost indiscernible $(1 \leq \mu \leq \tau < \kappa)$ if T has a saturated model of power κ which is the algebraic closure of an indiscernible set of μ-tuples. T is (μ, κ)-almost indiscernible iff T is (μ, τ^+)-almost indiscernible.

Any such theory T is superstable, stable in all cardinalities $\lambda \geq \tau$, and non-multidimensional. There is a pairwise orthogonal family $\mathcal{R}$ of regular types over M_τ, the τ-saturated model of cardinality τ, and a corresponding family $\mathcal{W}$ of types of maximal weight one sets over M_τ, such that any $N \succeq M_\tau$ is prime over M_τ and an independent family of realizations of types in $\mathcal{R}$, and is the algebraic closure of M_τ and an independent family of realizations of types in $\mathcal{W}$.

In a saturated free algebra in an uncountable language, the type of a basis element is well-defined. We conjecture that for an arbitrary such theory: it is totally transcendental, one-based, with quantifer elimination down to primitive formulas, and with finite Morley rank.

We have sharper results for theories of modules.