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Ill-posedness of the periodic nonlinear Schrödinger equation with third-order dispersion and Raman scattering term

We consider the nonlinear Schrödinger equation with third-order dispersion and derivative nonlinearity on the one-dimensional torus:

$$i\partial_t u + \partial_x^2 u + i\partial_x^3 u = c_1|u|^2u + ic_2\partial_x(|u|^2u) + i\gamma\partial_x(|u|^2)u, \qquad t \in \mathbb{R}, \quad x \in \mathbb{R}/2\pi\mathbb{Z},$$

where c_1, c_2 are real constants and γ is a complex constant. This equation is regarded as a mathematical model for the photonic crystal fiber oscillator, and the last term, with the coefficient γ having non-zero imaginary part, is related to the intrapulse Raman scattering effect, which is not negligible for ultrashort optical pulses. Without the Raman scattering term (i.e., $\mathrm{Im}\gamma = 0$), or for the non-periodic problem with any complex γ, the associated Cauchy problem is known to be locally well-posed in Sobolev spaces. We show that in the periodic setting the Raman scattering term causes ill-posedness (more precisely, non-existence of local-in-time solutions) of the Cauchy problem in Sobolev and Gevrey spaces. This talk is based on a joint work with Yoshio Tsutsumi (Kyoto University).