MICHAEL BRANNAN, Texas A&M University
Quantum permutations and their matrix models

A quantum permutation matrix is a square matrix P whose entries are orthogonal projections on a Hilbert space H with the property that the rows and columns of P sum to the identity operator on H. In the special case where H is the one dimensional Hilbert space, a quantum permutation matrix is simply an ordinary permutation matrix, and can be thought of as describing a symmetry of a finite set. In this talk I will explain how arbitrary quantum permutation matrices describe the "quantum symmetries" of finite sets. Putting all of these quantum permutation matrices together in a cohesive way yields the structure of a quantum group, which is commonly called the Quantum Permutation Group. Unlike the classical permutation groups, quantum permutation groups turn out to highly infinite and noncommutative objects – in many ways they behave algebraically like the C*- and von Neumann algebras associated to nonabelian free groups. Despite their inherent infiniteness, I will show how quantum permutation groups can still be well-approximated by finite-dimensional structures. In particular, these objects turn out to be residually finite as discrete quantum groups, and this residual finiteness can in fact be achieved using very simple finite-dimensional matrix models which I will describe. (Joint work with Alexandru Chirvasitu and Amaury Freslon.)