Nonlocal Interaction Models: From Collective Behaviour to Pattern Formation
Modèles d'interactions non locales : du comportement collectif à la formation de modèles
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ALETHEA BARBARO, Case Western Reserve University
A Kinetic Perspective on Fearful Crowds

Much of our life outside the home is spent in crowds: walking through the park on a warm summer day, watching a concert along with hundreds of other individuals, or racing to your classroom in the final minute before class. Buildings, parks, and concert halls are carefully designed to accommodate the masses. But what happens when disaster strikes? In this talk, we will introduce a new microscopic model for crowd dynamics in the presence of a strong emotion such as fear by including an emotional contagion component into the dynamics. We then derive a kinetic version of the model and explore its properties.

ANDREW BERNOFF, Harvey Mudd College
Agent-Based and Continuous Models of Locust Hopper Bands: Insights Gained Through the Lens of Dynamical Systems

Locust swarms pose a major threat to agriculture, notably in northern Africa, the Middle East and Australia. In the early stages of aggregation, locusts form hopper bands. These are coordinated groups that march in columnar structures that are often kilometers long and may contain millions of individuals. We report on two models for the formation of locust hopper bands. The first is a two-dimensional agent-based model (ABM) that incorporates intermittent motion, alignment with neighbors, and social attraction/repulsion, all behaviors that have been validated in experiments. Using a particle-in-cell computational method, we simulate swarms of up to a million individuals, which is several orders of magnitude larger than what has previously appeared in the locust modeling literature. We observe hopper bands in this model forming as an instability. Our model also allows homogenization to yield a system of partial integro-differential evolution equations. We identify a bifurcation from a uniform marching state to columnar structures, suggestive of the formation of hopper bands. The second is a one-dimensional ABM that introduces a resource (food) and includes foraging. Here homogenization yields a hyperbolic system of PDEs. Both the ABM and the PDEs manifest pulse solutions which are reflective of field observations. We reflect on the fact that both these models allow reductions that can be analyzed via methods from the study of dynamical system.

KATY CRAIG, University of California, Santa Barbara
Aggregation diffusion to constrained interaction: minimizers and gradient flows in the slow diffusion limit

Nonlocal interactions arise throughout the natural world, from collective dynamics in biological swarms to vortex motion in superconductors. Over the past fifteen years, there has been significant interest in aggregation diffusion equations, which consider the competing effects of nonlocal interactions and local diffusion. More recently, interest has also emerged in constrained aggregation equations, which consider the competition between nonlocal interactions and a hard height constraint on the density.

In joint work with Ihsan Topaloglu, we prove that aggregation diffusion equations converge to the constrained aggregation equation in the slow diffusion limit. As an application of this theoretical result, we adapt Carrillo, Craig, and Patacchini's blob method for diffusion to develop a numerical method for constrained aggregation equations, which we use to explore open conjectures in geometric shape optimization.

MARIA D'ORSOGNA, California State University at Northridge
Dynamics of neuroendocrine stress response: bistability, trauma and PTSD

The hypothalamic-pituitary-adrenal (HPA) axis is a neuroendocrine system that regulates numerous physiological processes. Disruptions are correlated with stress-related diseases such as PTSD and major depression. We characterize *normal* and *diseased* states of the HPA axis as basins of attraction of a dynamical system describing the main hormones regulated by the HPA axis. Our model includes hormonal self-upregulation, release, synthesis; interaction, delay and feedback mechanisms. External input is associated to psychological trauma, while parameter changes represent physiological damage. We show that transitions between the *normal* and *diseased* states may be induced solely by external input, with all physiological parameters unchanged, emphasizing the severe consequences of psychological trauma. We also find that the timing and duration of the traumatic event is an important determinant of if and how stress disorders will manifest. Finally, we propose mechanisms whereby exposure therapy may act to normalize downstream dysregulation of the HPA axis.

THOMAS HILLEN, University of Alberta
Non-local Models for Cellular Adhesion

Cellular adhesion is one of the most important interaction force between cells and other tissue components. In 2006, Armstrong, Painter and Sherratt introduced a non-local PDE model for cellular adhesion, which was able to describe known experimental results on cell sorting and cancer growth. While the numerical implementation leads to nice results, the analysis of this non-local model is challenging. In this talk I will present a random walk derivation of the Armstrong-Painter-Sherratt adhesion model, I will discuss local and global existence of solutions, the underlying bifurcation structure of steady states, and a biologically accurate definition of non-local boundary conditions. (joint work with A. Buttenschoen, K. Painter, A. Gerisch, M. Winkler).

HUI HUANG, Simon Fraser University
On self-organized dynamics over domains with boundaries

Systems of interacting particles are widely used to establish different mathematical models describing collective behaviors of organisms and social aggregations. In this talk, we focus on such self-organized dynamics over domains with boundaries, which are commonly involved in realistic physical settings. For example, the boundary can be an obstacle in the environment, such as a river or the ground. Our research contributions include the mean-field limit for the particle system with reflecting boundary condition and the zero-diffusion limit from the aggregation-diffusion model to the plain aggregation model.

DAVID IRON, Dalhousie
Delay and Spike Dynamics in the Gierer-Meinhardt Equations

We consider the effect of Delay in the reaction terms of the Gierer-Meinhardt system. We will consider delay in both the auto-catalytic terms as well as the inhibitory terms. The system of delayed partial differential equations is reduced to an ordinary delay differential equation for the motion of spike solution. We consider stability and possible Hopf bifurcations.

THEODORE KOLOKOLNIKOV, Dalhousie University
Spikes and vortices in inhomogeneous environments

I will discuss spike distribution in reaction-diffusion systems with space-dependent parameters. This involves two levels of reduction: first, we use by-now standard asymptotic techniques to derive a system of reduced equations for spike centers and height. Second, we take a continuum limit of the reduced system, resulting in novel system of nonlocal PDE's. Finally the resulting PDE's are solved asymptotically to yield an effective description of spike density. As a bonus, we obtain instability thresholds for N spikes in one dimension. We also generalize these results in two dimensions. Several open questions will also be presented.

XIN YANG LU, Lakehead University
Regularity of cells in crystallization of the second moment energy in 3D

Centroidal Voronoi Tessellations (CVT) are tessellations using Voronoi regions of their centroids. CVTs are useful in several fields, such as data compression, optimal quadrature, optimal quantization, clustering, and optimal mesh generation. Many patterns seen in nature are closely approximated by a CVT. Gersho's conjecture, which states that there exists an asymptotically optimal CVT whose Voronoi regions are all rescaled copies of the same polytope. Straightforward in 1D, and proven in 2D, Gersho's conjecture is still open for higher dimensions. One of the main difficulties is that Gersho's conjecture is a strongly nonlocal, infinite dimensional minimization problem (even in 3D). In this talk we will present some recent results which reduce Gersho's conjecture to a local, finite dimensional problem in 3D. Joint work with Rustum Choksi.

ALPAR MESZAROS, University of California, Los Angeles
Degenerate nonlinear diffusion equations with discontinuous nonlinearities

In this talk, I will consider Cauchy problems for degenerate diffusion equations of the form $\partial_t \rho - \Delta\varphi(\rho) = 0,\ \rho(0,\cdot) = \rho_0$ on smooth bounded domains of $\mathbb{R}^d$, where ρ_0 is a given probability measure. The increasing nonlinearity $\varphi : [0,+\infty) \to \mathbb{R}$ is supposed to have a discontinuity at some $s_0 \in (0,+\infty)$. Such models arise in mathematical biology describing so-called *self-organized criticality* phenomena. To show the well-posedness of such problems, for a large class of nonlinearities, we rely on its gradient flow formulation in the space of probability measures equipped with the so-called L^2-*Wasserstein distance*. We show that in general a 'critical zone', $\{\rho = s_0\}$ emerges, and the problem gives rise to a three-phase Stefan-type problem. We will show that a *pressure term* appears in the precise description of the solution, which is concentrated on the critical zone. By this observation, we can make a link to recent models on congested crowd dynamics. The talk is based on a joint work with Dohyun Kwon (UCLA).

FRANCESCO PATACCHINI, Carnegie Mellon University
The interaction equation near attracting manifolds

We study the existence and uniqueness of solutions to the interaction equation on manifolds, where the velocity is the projection of the Euclidean velocity onto the tangent space of the manifold. For the uniqueness, a stability estimate can be obtained under the assumption that the manifold be of positive reach. For the existence, we approximate the interaction energy on the manifold by the classical full-space interaction energy to which we add a confinement part which is proportional to the distance to the manifold and which blows up as a parameter epsilon tends to 0. This ensures that any mass living near the manifold gets attracted towards it as epsilon vanishes. We use the Sandier-Serfaty approach to show that, in the limit as epsilon goes to 0, the gradient flow in the full space converges to the gradient flow on the manifold. This is ongoing work with D. Slepcev.

NANCY RODRIGUEZ-BUNN, CU Boulder
Birth-jump process in plan dynamics

In this talk I will introduce and provide an analysis of a discrete agent-based-model, based on a birth-jump process, to model plant dynamics. The system exhibits wave-like solutions. In order to rigorously analyze this traveling-wave phenomena I will present the diffusion-limit of the discrete model and prove the existence of traveling wave solutions (sharp and continuously differentiable) assuming a logarithmic-type growth. Furthermore, I will discuss a variational speed for the minimum speed of the waves and perform numerical experiments that confirm our results.

XIAOCHUAN TIAN, The University of Texas at Austin
Nonlocal models with heterogeneous localization and their application to seamless local-nonlocal coupling

Motivated by recent development on nonlocal mechanical models like peridynamics, we consider nonlocal integral models with a spatially varying horizon that allows the finite range of nonlocal interactions to be position-dependent. In particular, we focus on linear variational problems of such nonlocal models with heterogeneous localization on co-dimension one interfaces. The well-posedness is established for variational problems. We also study their seamless coupling with local models. In addition, we present numerical studies of the nonlocal models and local-nonlocal coupled models based on the asymptotically compatible Galerkin finite element discretizations.