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Non-Local Cell Adhesion Models: Steady States and Bifurcations

In both normal tissue and disease states, cells interact with one another, and other tissue components using cellular adhesion proteins. These interactions are fundamental in determining tissue fates, and the outcomes of normal development, and cancer metastasis. Traditionally continuum models (PDEs) of tissues are based on purely local interactions. However, these models ignore important nonlocal effects in tissues, such as long-ranged adhesion forces between cells. For this reason, a mathematical description of cell adhesion had remained a challenge until 2006, when Armstrong et. al. proposed the use of an integro-partial differential equation (iPDE) model. Since then this approach has proven popular in applications to embryogenesis (Armstrong et. al. 2009), zebrafish development (Painter et. al. 2015), and cancer modelling (e.g. Painter et. al. 2010, Domschke et. al. 2014, Bitsouni et. al. 2018). While popular, the mathematical properties of this non-local term are not yet well understood.

In this talk, I will present our recent results of a study of the steady-states of a non-local adhesion model on an interval with periodic boundary conditions. The significance of the steady-states is that these are observed in experiments (e.g. cell-sorting). Combining global bifurcation results pioneered by Rabinowitz, equivariant bifurcation theory, and the mathematical properties of the non-local term, we obtain a global bifurcation result for the branches of non-trivial solutions. Using the equation's symmetries the solutions of a branch are classified by the derivative's number of zeros. We further show that the non-local operator's properties determine whether a sub or super-critical pitchfork bifurcation occurs.