Explicit Methods in Arithmetic Geometry
Méthodes explicites en géométrie arithmétique

(Org: **Nils Bruin** (Simon Fraser University) and/et **Michael Jacobson** (University of Calgary))

JEFF ACHTER, Colorado State University
Arithmetic moduli for lattice-polarized K3 surfaces

or, *Three views of six points*.

Several natural complex configuration spaces admit surprising uniformizations as arithmetic ball quotients. I'll use recent advances in the theory of K3 surfaces to construct arithmetic period maps which explain these uniformizations.

MICHAEL BENNETT, University of British Columbia
Shifted powers in Lucas-Lehmer sequences

We develop a general framework for finding all perfect powers in sequences derived by shifting non-degenerate quadratic Lucas-Lehmer binary recurrence sequences by a fixed integer. By combining this setup with bounds for linear forms in logarithms and results based upon the modularity of elliptic curves defined over totally real fields, we are able to answer a question of Bugeaud, Luca, Mignotte and Siksek by explicitly finding all perfect powers of the shape $F_k \pm 2$ where F_k is the k-th term in the Fibonacci sequence. This is joint work with Vandita Patel and Samir Siksek.

NATHAN GRIEVE, Michigan State University
Around the Riemann-Roch Theorem for Abelian varieties

In this talk, I will explain how the Riemann-Roch Theorem for divisors on an Abelian variety A is related to the reduced norms of the Wedderburn components of its endomorphism algebra. Motivated by this result, I will also mention more recent observations, building on work of Atiyah, Brion, Mukai and others, which pertain to Severi-Brauer varieties over A. For example, the Brauer group of A can be interpreted through the concept of theta groups.

BETH MALMSKOG, Colorado College
Solving S-unit equations in Sage and Applications

Let K be a number field. Many finiteness results in number theory and arithmetic geometry rely on the fact, due to Thue, Siegel, Mahler, and Lang, that for any non-zero a and b in K, the equation ax+by=1 has only finitely many solutions in any finitely generated multiplicative subgroup of K. A particularly useful consequence of this result is that the equation x+y=1 has finitely many solutions in the S-units of K. Work of Baker, Yu, de Weger, Smart, and many others resulted in practical algorithms for determining these solutions. However, until now, there has been no publicly available implementation of these algorithms in a computer algebra system. A group consisting of Alejandra Alvarado, Angelos Koutsianas, me, Chris Rasmussen, Christelle Vincent, and Mckenzie West has recently implemented functions in Sage to solve the S-unit equation for general K and S. In this talk, I will outline the algorithms, discuss current computational limitations, and share a few applications (and potential applications) in algebraic curves and number theory.

BRETT NASSERDEN, University of Waterloo
Explicit calculations with a moduli space of abelian surfaces

Th moduli space of principally polarized abelian surfaces with a full level 3 structure can be describe by the threefold B : $y_0(y_0^3+y_1^3+y_2^3+y_3^3+y_4^3)+3y_1y_2y_3y_4=0$. We will explain how to use the geometry of B to explicitly construct a family of smooth genus 2 curves, and discuss the geometric origin of the level 3 structure on these curves. This is joint work with Nils Bruin.

JENNIFER PAULHUS, Grinnell College
Jacobian variety decompositions

Jacobian varieties which can be factored into the product of elliptic curves have interesting applications to rank and torsion questions. Given a curve X with automorphism group G, idempotent relations in the group ring $\mathbb{Q}[G]$ lead to decompositions of the Jacobian of X. In this talk we discuss some recent results obtained from these techniques. Particularly, new computational advances and the study of intermediate covers allow us to determine these decompositions for curves in high genus, and we use that to find many new examples of completely decomposable Jacobians, including families of such curves.

ARI SHNIDMAN,
Selmer groups of genus 2 Jacobians with root 3 level structure

We give an explicit parameterization of the universal genus two curve with "square root three" multiplication and level structure. Using this, we produce a large family of Jacobians having the property that a positive proportion of their quadratic twists have non-trivial Tate-Shafarevich groups. We also construct a universal family of abelian surfaces with "fake real multiplication," and study the Mordell-Weil ranks of their quadratic twists. Joint work with Nils Bruin and Victor Flynn.

ADAM TOPAZ, University of Alberta
Reconstructing Function Fields from their ℓ-adic Cohomology

This talk will present some recent work in progress which shows that the function field of a higher-dimensional variety is determined, up-to isomorphism, from its ℓ-adic cohomology ring, when it is endowed with the Galois action of a "sufficiently global" base field. A key step in this result, which may be of independent interest, is the explicit determination of the divisorial vauations of the function field in question, and the cohomology of their residue fields, using the given Galois-theoretical information. A comparison with Bogomolov's programme and the Bogomolov-Pop conjecture in birational anabelian geometry will also be discussed.

COLIN WEIR, Tutte Institute for Mathematics and Computing
Classifying the p-torsion of Jacobians and Pryms

The distinction between elliptic curves being either supersingular or ordinary is essentially a distinction between their respective p-torsion group schemes. In higher dimensions more than those two possibilities can occur; there are 2^g possible isomorphism classes of p-torsion group schemes of dimension g in characteristic p. In this talk we will present an algorithm which, given a curve in characteristic p, will compute the isomorphism type of the p-torsion of its Jacobian. We will also discuss a Magma package that efficiently implements this algorithm together with several other useful methods. In particular, we will show how these techniques can be used to classify the p-torsion of Prym varieties as well. This is joint work with Mark Bauer.