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On representing matroids by matrices and graphs: two sides of the same coin

Matroids provide a combinatorial abstraction of the notion of dependence, analogous to the way groups abstract symmetry and topologies abstract continuity. Two fundamental classes of matroids are those that can be represented as a matrix over a field and those that can be represented as a biased graph. Those matroids with a representation of both types have a central role in matroid structure theory.

Let M be a 3-connected matroid and let $\mathbb{F}$ be a field. Let A be a matrix over $\mathbb{F}$ representing M and let $(G, \mathcal{B})$ be a biased graph representing M. Is there any relationship between the matrix and the graph? Yes! A is projectively equivalent to a canonical matrix representation of M arising from G as a gain graph over the additive or multiplicative group of $\mathbb{F}$. Further, the projective equivalence classes of matrix representations of M are in one-to-one correspondence with the switching equivalence classes of gain graphs arising from $(G, \mathcal{B})$.

This is joint work with Daniel Slilaty.