
Financial Mathematics
Mathématiques financières
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ABEL CADENILLAS, University of Alberta
Optimal Stochastic Control of the Government Stabilization Fund

To mitigate the negative consequences of a crisis, an instrument of fiscal policy is the government stabilization fund, which is a mechanism to save money in the good economic times to be used in the bad economic times. We present a model to study the optimal stochastic control of a government stabilization fund.

JOE CAMPOLIETI, Wilfrid Laurier University
Solvable Diffusion Models with Linear and Mean-Reverting Nonlinear Drifts

We present an extension to a method for constructing solvable diffusions. This gives rise to new solvable models that are divided into two main classes; the first is specified by having a linear drift with various resulting nonlinear diffusion coefficient functions, while the second class allows for several specifications of (generally) nonlinear diffusion coefficient functions with resulting nonlinear drift function. The first class of models is useful for pricing equity and foreign exchange (FX) options in finance, while the second class contains new models that are mean-reverting and are applicable to pricing interest-rate and other path-dependent derivatives such as volatility index (VIX) options. As specific examples of the first class, we present explicit results for two new families of solvable models. For the second class, we give examples of new solvable nonlinear mean-reverting processes and derive closed-form integral formulas for conditional expectations of certain functionals. In particular, we derive a new closed-form analytical formula for the Laplace transform with respect to the strike price of a standard call VIX option. We then succeed in Laplace inverting the expression to obtain exact VIX call option prices with realistic implied volatilities with respect to strike and maturity. Lastly, we accurately calibrate another of our new models to USD/EUR FX option market data exhibiting pronounced implied volatility smiles across several strikes and maturities.

TAHIR CHOULLI, University of Alberta
Structure Condition In Informational Markets

It has been understood that the existence of Markowitz' optimal portfolio and the solution to the local-risk minimization problem are intimately related to some mathematical structures on the underlying assets price processes. These structures are known, in the literature, as "Structure Condition". In this talk, we consider a market model (initial market model) fulfilling these structures, and an arbitrary random time that is not a stopping time with respect to the flow of the information generated by the initial market. This random time can model the default time of a firm, the death time of an insured, or any time occurrence of an event that might affect the market some how. By adding —progressively through time— the information about this random time as it occurs, those structures may fail and hence the optimal portfolio/strategy will fail to exist. Our aim is to address the question of how the incorporation of this random time will affect these structures from different perspectives. Our analysis allowed us to conclude that under some mild assumptions on the market model and the random time, these structures will remain valid on the one hand. On the other hand, we describe the random time models for which these structure conditions are preserved for any market model.

CHRIS FREI, University of Alberta
Managing Counterparty Risk in Over-the-counter Markets

We study how banks manage their default risk before bilaterally negotiating the quantities and prices of over-the-counter (OTC) contracts resembling credit default swaps (CDSs). We show that the costly actions exerted by banks to reduce their default probabilities are not socially optimal. Depending on the imposed trade size limits, risk-management costs and sellers'

bargaining power, banks may switch from choosing default risk levels above the social optimum to reducing them even below the social optimum. We use a unique and comprehensive data set of bilateral exposures from the CDS market to test the main model implications on the OTC market structure: (i) intermediation is done by low-risk banks with medium credit exposure; (ii) all banks with high credit exposures are net buyers of CDSs, and low-risk banks with low credit exposures are the main net sellers; and (iii) heterogeneity in post-trade credit exposures is higher for riskier banks and smaller for safer banks. The talk is based on joint work with Agostino Capponi (Columbia University) and Celso Brunetti (Federal Reserve Board).

NIUSHAN GAO, Ryerson University

A general representation theory for risk measures

For a coherent risk measure $\rho : L^\infty \rightarrow \mathbb{R}$, Delbaen (2002) proved that ρ can be represented as the worst expectation over a class of probabilities whenever it has the Fatou property. Lately, it has been asked whether Delbaen's representation theorem holds on more general model spaces containing unbounded positions, specifically, Orlicz spaces. In this talk, we present a comprehensive investigation on this problem. We characterize the Orlicz spaces over which the theorem holds, which shows, in particular, that the theorem fails on a fairly large class of Orlicz spaces, countering to the general positive belief. Next, we show that the theorem holds on general Orlicz spaces if the risk measure possess additional properties, e.g., law-invariance or surplus-invariance.

This talk is based on joint papers with D. Leung, C. Munari, F. Xanthos.

ANASTASIS KRATSIOS, Concordia University

Geometric Learning and Non-Euclidean Filtering in Finance

We introduce a method for incorporating relevant geometric information into a broad class of filtering and learning algorithms. Our central theorems illustrate how we may computably approximate the non-Euclidean filtering problem to arbitrary precision and how we can increase the accuracy of any learning algorithm by incorporating the relevant geometric information into a large class of filtering and machine learning algorithms. Our applications focus on exploiting the geometry of the stochastic volatility models and the shape of the forward-rate curve.

ALEXEY KUZNETSOV, York University

Lognormal convolutions and their applications

Computing the distribution of a sum of independent lognormally distributed random variables is a hard problem that has many important applications in Actuarial Science. I will present a new algorithm for approximating the sums of independent and lognormally distributed random variables; this algorithm actually works for a more general class of Generalized Gamma Convolutions. By merging tools from probability theory and numerical analysis, we are able to compute the cumulative distribution functions of the just-mentioned sums with any desired precision. Our algorithm is fast and can tackle equally well sums with just a few or thousands of summands. The effectiveness of the new method will be illustrated in the contexts of the individual and collective risk models, aggregate economic capital determination, and economic capital allocation. This talk is based on joint work with Ed Furman and Dan Hackmann.

ROMAN MAKAROV, Wilfrid Laurier University

Stochastic Modelling of Assets with Missing Pricing Data

We present new multivariate diffusion and jump-diffusion models for dealing with financial securities that have missing or asynchronous pricing data. The models allow us to analyze a portfolio that combines a high activity asset such as a market index (or an exchange-traded fund tracking a market index) and several low-activity assets. The models are constructed in such a way that low-activity assets correlate with each other only through the high-activity asset price process. For the calibration of models, we estimate parameters for a high-activity asset first and then estimate parameters for each low-activity asset conditional on the parameters for the high-activity asset. In doing so, we use the maximum likelihood method. For the model that is based on a multivariate geometric Brownian motion, we derive analytical pricing formulas for basket options.

ROGEMAR MAMON, University of Western Ontario

A higher-order Markov chain-modulated model for electricity spot-price dynamics

As electricity is a non-storable commodity, its price is extremely sensitive to changes in supply and demand. Electricity-price evolution exhibits pronounced mean reversion and cyclical patterns, possesses extreme volatility and relatively frequently occurring spikes, and manifests presence of memory property. These observed features necessitate the development of models aimed to simultaneously capture such price characteristics for forecasting, risk management, and valuation of electricity-driven derivatives. This work tackles the modelling and estimation problems under a new paradigm that integrates the deterministic calendar seasons and stochastic factors governing electricity prices. The de-seasonalised component of our proposed model has both the jump and mean-reverting properties to account for spikes and periodic cycles alternating between lower price returns and compensating periods of higher price returns. The parameters of the de-seasonalised model components are also modulated by a higher-order hidden Markov chain (HOHMC) in discrete time. This provides a mechanism to extract latent information from historical data. The HOHMC's state is interpreted as the "state of the world" resulting from the interaction of various forces impacting the electricity market. Filters are developed to generate optimal estimates of HOHMC-relevant quantities using the observation process, and these provide online estimates of model parameters. Empirical demonstrations, using daily electricity spot prices, compiled by the Alberta Electric System Operator, show that our HOHMM approach has considerable merits in terms of price data fitting and forecasting metrics. Implications of our model to the pricing of an electricity forward contract are also examined. This is joint work with H. Xiong.

ADAM METZLER, Wilfrid Laurier University

State Dependent Correlations and Economic Capital

Under Basel, banks must use the so-called risk-weight function to determine economic capital. The function is based on a simple and intuitive model developed by Vasicek (2002) - a model that makes several unrealistic assumptions but appears to retain its popularity due to its intuitive appeal. One of these assumptions is that correlations are independent of the state of the economy, which is in stark contrast to empirical evidence for most asset classes. In this talk we generalize the Vasicek model to allow for correlations that tend to rise during adverse economic scenarios. The model allows the user to control the degree of state dependence, by which we mean the likelihood that correlations are high when the economy is bad. Economic capital is (unsurprisingly) highly sensitive to the degree of state dependence, and empirical evidence (using Federal Reserve data on delinquency rates) indicates that in practice the degree of state dependence is remarkably high.

RADU ILIE MITRIC, Université Laval

Properties of risk measures inspired from a ruin model with interest

Recent research has established some connections between ruin theory for the classical Cramer-Lundberg risk model and several risk measures with respect to the stochastic ordering of claim severities. We explore some extensions. For the classical ruin model we investigate some properties of a risk measure derived from the Laplace transform of the ruin time. Then, for an enhanced model that include the effect of the interest rate, we study a risk measure derived from the so called expected area in red (expected area of the negative part of a risk process).

TRAIAN PIRVU, McMaster University

Longevity bond pricing in equilibrium

We consider a partial equilibrium model for pricing a longevity linked bond in a model with stochastic mortality intensity that affects the income of economic agents. The agents trade in a risky financial security and in the longevity linked bond in order to maximize their utilities. Agent's risk preferences are of monetary type and are described by BSDEs (backward stochastic differential equations). The endogenous equilibrium bond price is characterized by a BSDE. By using Clark-Haussmann formula, we prove that our longevity bond completes the market.

MARK REESOR, Wilfrid Laurier University

Capital Structure Effects on the Prices of Equity Call Options: First Passage Time Approach

Capital structure models treat equity as a call option on firm value and hence traded equity options are viewed as compound options (CO) on firm value. Using the CO interpretation, recent work (Geske et al 2016) has shown that prices of traded equity options depend on a firm's capital structure. This work is done in the Merton framework in which default and liquidation of the firm is allowed only at one specific future date. In our work, we extend the CO analysis to the first-passage time (FPT) framework in which default occurs the first time that firm value breaches a barrier. We derive valuation equations and show that the FPT framework provides greater flexibility in fitting option-implied volatility curves as compared to the Merton framework. Case studies using actual equity option prices show the FPT approach significantly outperforms the Merton approach across a range of option moneynesses and maturities. As part of the calibration, we obtain market-implied leverage and firm volatility, which can be used in other corporate finance applications.

This is joint work with Xinghua (Alan) Zhou of Western University.

KRISTINA SENDOVA, University of Western Ontario

Capturing simultaneous claim occurrences

This research is data-driven. Our goal is to take into consideration that claim data is recorded in a way that most claims occur simultaneously with other claims. As a result, we are searching for an appropriate claim-counting model that would reflect this important feature of the data while allowing for further analysis of risk measures that are associated with the insurance company's business.

FOIVOS XANTHOS, Ryerson University

Spanning of options and lattice theory

Ross (1976) has shown, in a finite-state framework, how options can complete financial markets. In this talk, we will study this problem in markets with infinitely many states and establish a connection with the theory of vector lattices. As a by-product of our results, we establish a generalization of the Kreps-Yan theorem and show that a pricing rule for vanilla options can be uniquely extended by arbitrage to a pricing rule on all derivative assets. This talk is based on joint work with Niushan Gao.

FENGHAO YANG, Yor University

Pricing analogues of first-to-default options under uncertain correlation.

We study correlation uncertainty in Credit Risk. The goal is to price analogues of first-to-default options under the assumption that the assets follow correlated stochastic processes with known marginal distributions and unknown dependence structure. We solve this problem using tools from Stochastic Analysis and Optimal Control Theory. We provide explicit solutions in some specific examples and numerical approximations in the more general case. This is a joint work with Thomas Salisbury and Alexey Kuznetsov.