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Reducibility of Quasi-Periodic Linear KdV Equation

We consider the following one-dimensional, quasi-periodically forced, linear KdV equations

$$u_t + (1 + a_1(\omega t, x))u_{xxx} + a_2(\omega t, x)u_{xx} + a_3(\omega t, x)u_x + a_4(\omega t, x)u = 0$$

under the periodic boundary condition $u(t, x+2\pi) = u(t, x)$, where ω's are frequency vectors lying in a bounded closed region $\Pi_* \subset R^b$ for some $b > 1$, $a_i : T^b \times T \to R$, $i = 1, \cdots, 4$, are bounded above by a small parameter $\epsilon_* > 0$ under a suitable norm, real analytic in $\phi \in T^b$ and sufficiently smooth in $x \in T$, and a_1, a_3 are even, a_2, a_4 are odd. Under the real analyticity assumption of the coefficients, we show that there exists a Cantor set $\Pi_{\epsilon_*} \subset \Pi_*$ with $|\Pi_* \setminus \Pi_{\epsilon_*}| = O(\epsilon_*^{\frac{1}{100}})$ such that for each $\omega \in \Pi_{\epsilon_*}$, the corresponding equation is smoothly reducible to a constant-coefficients one. This problem is closely related to the existence and linear stability of quasi-periodic solutions in a nonlinear KdV equation.