GEORGE ELLIOTT, University of Toronto
The Rieffel projection via groupoids (joint work with Dickson Wong)

The Rieffel projection is both not quite canonical and, perhaps, a little hard to understand. The Rieffel Hilbert module, from which it derives, is perhaps canonical, but is, on the face of it, especially when one considers the algebra-valued inner product, also not that easy to understand.

It turns out that a simple extension of the well-known groupoid underlying the rotation algebra, which itself does not yield a non-trivial projection in the algebra since its object space is connected, has a disconnected space of objects. One of its clopen components corresponds to the unit of the rotation algebra, and its complement corresponds to a projection in a larger, Morita equivalent, C*-algebra with the same K_0-class as the Rieffel projection.

This groupoid, consisting of a disconnected version of the cut-down of the groupoid of the Kronecker flow to the figure eight consisting of the union of the two generating circles on the torus, can also be used to construct the Rieffel module directly. One notices that it has a (clopen) copy of the real line on which the cut-down subgroupoid corresponding to the rotation algebra acts, and then the natural module action of the algebra of continuous functions of compact support on the groupoid on the linear space of continuous functions of compact support on the line, arising from (finite sum) convolution, constitutes on completion the Rieffel Hilbert module over the rotation C*-algebra. (This construction yields in a natural way that the module is finitely generated and projective.)