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A stable arithmetic regularity lemma in finite-dimensional vector spaces over fields of prime order

In this talk we present a stable version of the arithmetic regularity lemma for vector spaces over fields of prime order. The arithmetic regularity lemma for $\mathbb{F}_p^n$ (first proved by Green in 2005) states that given $A \subseteq \mathbb{F}_p^n$, there exists $H \leq \mathbb{F}_p^n$ of bounded index such that A is Fourier-uniform with respect to almost all cosets of H. In general, the growth of the index of H is required to be of tower type depending on the degree of uniformity, and must also allow for a small number of non-uniform elements. Our main result is that, under a natural stability theoretic assumption, the bad bounds and non-uniform elements are not necessary. Specifically, we present an arithmetic regularity lemma for k-stable sets $A \subseteq \mathbb{F}_p^n$, where the bound on the index of the subspace is only polynomial in the degree of uniformity, and where there are no non-uniform elements. This result is a natural extension to the arithmetic setting of the work on stable graph regularity lemmas initiated by Malliaris and Shelah. This is joint work with Julia Wolf.