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Some recent results on Q-polynomial (cometric) association schemes

Let X be a finite set of size v and let $\mathcal{R} = \{R_0, \dots, R_d\}$ be a partition of $X \times X$ into $d + 1$ symmetric binary relations with R_0 equal to the identity relation on X . The pair (X, \mathcal{R}) is called a *symmetric d -class association scheme* provided there exist integers p_{ij}^k ($0 \leq i, j, k \leq d$) such that whenever $a, b \in X$ with $(a, b) \in R_k$, we have $|\{c \in X : (a, c) \in R_i, (c, b) \in R_j\}| = p_{ij}^k$. If A_i is the 01-matrix with rows and columns indexed by X and (a, b) -entry equal to one iff $(a, b) \in R_i$, then the *Bose-Mesner algebra* of the association scheme is given by $\mathbb{A} = \langle A_0, \dots, A_d \rangle$. This matrix algebra admits a basis $\{E_0, E_1, \dots, E_d\}$ satisfying $E_i E_j = \delta_{i,j} E_i$. The Schur (or entrywise) product of any two of these idempotents belongs to \mathbb{A} so there exist scalars q_{ij}^k ($0 \leq i, j, k \leq d$) satisfying

$$E_i \circ E_j = \frac{1}{v} \sum_{k=0}^d q_{ij}^k E_k. \quad (1)$$

An association scheme (X, \mathcal{R}) is *cometric* (or *Q-polynomial*) if there exists an ordering E_0, \dots, E_d with respect to which $q_{ij}^k = 0$ whenever $k > i + j$, and $q_{ij}^k \neq 0$ whenever $k = i + j$. In this talk, we explore the spherical code formed by the columns of E_1 and establish inequalities for certain valencies of the (regular) graphs (X, R_i) and the class number d in terms of its rank.