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Intersections of Cantor Sets and Self-Similarity

We study finite intersections of multiplicative translates of p-adic Cantor sets. For $p = 3$, this is motivated by a problem of Erdős on the base 3 expansions of powers of 2. Consider the discrete dynamical system on the 3-adic integers $\mathbb{Z}_3$ given by multiplication by 2. The exceptional set $\mathcal{E}(\mathbb{Z}_3)$ is defined to be the set of all elements of $\mathbb{Z}_3$ whose forward orbits under this action intersect the 3-adic Cantor set $\Sigma_{3,\bar{2}}$ (of 3-adic integers that omit the digit 2) infinitely many times. This set is conjectured to have Hausdorff dimension 0, and attempts to prove this conjecture have led to the study of many interesting families of intersections of Cantor sets. These intersection sets are fractals whose points have 3-adic expansions describable by labeled paths in a finite automaton, whose Hausdorff dimension is exactly computable and is of the form $log_3(\beta)$ where β is a real algebraic integer.

The theoretical framework that we have developed to study intersections of Cantor sets, including the idea of a path set and of p-adic path set fractals, has found application to the study of multi-layer cellular networks. I will also discuss current work using this framework to study the self-similarity of intersections and unions of translations of Cantor sets.