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On the Circuit Diameter Conjecture

A key concept in optimization is the combinatorial diameter of a polyhedron. From the point of view of optimization, we would like to relate it to the number of facets f and dimension d of the polyhedron. In the seminal paper of Klee and Walkup, the Hirsch conjecture, that the bound is $f - d$, was shown to be equivalent to several seemingly simpler statements, and was disproved for unbounded polyhedra through the construction of a particular 4-dimensional polyhedron with 8 facets. The Hirsch bound for polytopes was only recently narrowly exceed by Santos.

We consider analogous questions for a variant of the combinatorial diameter called the circuit diameter. In this variant, paths are built from the circuit directions of the polyhedron, and can travel through the interior. We are able to recover the equivalence results that hold in the combinatorial case. Further, we show that validity of the circuit analogue of the non-revisiting conjecture for polytopes would imply a linear bound on the circuit diameter of all unbounded polyhedra. Finally, we prove a circuit version of the 4-step conjecture. These results offer some hope that the circuit version of the Hirsch conjecture may hold, even for unbounded polyhedra.

Our methods require adapting the notion of simplicity to work with circuits. We show that it suffices to consider such circuit simple polyhedra for studying circuit analogues of the Hirsch conjecture, and use this to prove the equivalences of the different variants.

This is joint work with Steffen Borgwardt and Timothy Yusun.