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On the Bivariate Erdős-Kac Theorem and Correlations of the Mobius Function

Let ω denote the arithmetic function that counts the number of distinct prime factors of an integer, and let $d \geq 1$. We develop a bivariate probabilistic model to study the joint distribution of the deterministic vectors $(\omega(n), \omega(n+d))$ with $n \leq x$ as $x \rightarrow \infty$, where n and $n+d$ are both required to be squarefree. The following three results are applications:

i) We establish a quantitative version of the bivariate Erdős-Kac theorem, a result first proven in a non-quantitative way by W. LeVeque, on a proper subset of \mathbb{N} .

ii) We give two partial results in the direction of a conjecture of Chowla on binary correlations of the Möbius function. Let $\mu_z(n) := \mu^2(n)(-1)^{\omega(n;z)}$, where $\omega(n;z)$ is the number of distinct primes $p|n$ with $p \leq z$. Then as long as $\frac{\log x}{\log z} \rightarrow \infty$ as $x \rightarrow \infty$, $\sum_{n \leq x} \mu_z(n)\mu_z(n+1) = o(x)$.

In a related way, we show that if $\mu(n;u) := \mu^2(n)e^{iu\omega(n)}$ then provided that $u = o(1)$ and $u\sqrt{\log_2 x} \rightarrow \infty$ as $x \rightarrow \infty$ then $\sum_{n \leq x} \mu(n;u)\mu(n+1;u) = o(x)$.

iii) We finally prove a partial result in the direction of a conjecture of Erdős, Pomerance and Sarközy on the order of magnitude of the number of $n \leq x$ such that $\tau(n) = \tau(n+1)$.