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High order mimetic discretizations

In this work we will present the High Order Mimetic Discretization Framework and discuss two fundamental aspects for the construction of structure-preserving discretizations: (i) the definition of the discrete degrees of freedom of physical field quantities, and (ii) the formulation of the physical field laws.

For the first, we will introduce geometric degrees of freedom, which are associated to geometric objects (points, lines, surfaces and volumes), and then establish their relation to differential forms. It will be shown that we can construct discrete polynomial function spaces of arbitrary degree associated to these geometric degrees of freedom. These function spaces constitute a discrete de Rham complex:

$$\mathbb{R} \longrightarrow V_h^0 \subseteq H(\nabla,\Omega) \xrightarrow{\nabla} V_h^1 \subseteq H(\nabla\times,\Omega) \xrightarrow{\nabla\times} V_h^2 \subseteq H(\nabla\cdot,\Omega) \xrightarrow{\nabla\cdot} V_h^3 \subseteq L^2(\Omega) \to 0\,.$$

In this way it is possible to exactly discretize topological equations even on highly deformed meshes. All approximation errors are included in the constitutive equations, which are encoded in the Hodge-$\star$ operator. This leads to discretizations that exactly preserve the divergence free constraint of velocity fields in incompressible flow problems and of magnetic fields in electromagnetic problems, for example.

For the second, the Navier-Stokes equations will be used as an example and we will show that although at the continuous level all equivalent formulations are equally good, at the discrete level, the choice of a particular formulation has a fundamental impact on the conservation properties of the discretization.

These ideas will be applied to the solution of: e.g. Poisson equation, Maxwell eigenvalue problem, Darcy flow, fusion plasma equilibrium and Navier-Stokes equations.