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On the Skellam model with time delay and non-zero drift

In this talk we consider the following initial value problem

$$\begin{cases} (\partial_t - \Delta_x + w \cdot \nabla_x + \alpha I)u(t, x) = \beta u(t - \tau, x), & t \in \mathbb{R}_+, x \in \mathbb{R}^n \\ u(t, x) = \phi(t, x) \in C([-\tau, 0]; L^1(\mathbb{R}^n)), & t \in [-\tau, 0], x \in \mathbb{R}^n. \end{cases}$$

The question we are interested in is the following. Under which conditions on parameters τ, w, α, β , is the trivial solution $u = 0$ stable?

In the context of population dynamics, this initial value problem can be viewed as a model of a population undergoing Malthusian growth and spreading by a random diffusion with the drift w . The growth is characterized by a death rate α , birth rate β and a gestation/maturation period τ . The problem is: given coefficients α, β, w, τ determine if the population invade the habitat or goes extinct.

In the drift-free case, a complete solution was given by Travis and Webb. We will show that in some cases the relation between α and β plays the dominant role in the extinction of the population. However, in the opposite cases, the drift can help the population to survive.