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Stronger Rolle's Theorem for Complex Polynomials

Every Calculus student is familiar with the classical Rolle's theorem stating that if a real polynomial p satisfies $p(-1) = p(1)$, then it has a critical point in $(-1, 1)$. In 1934, L. Tschakaloff strengthened this result by finding a minimal interval, contained in $(-1, 1)$, that holds a critical point of every real polynomial with $p(-1) = p(1)$, up to a fixed degree. In 1936, he expressed a desire to find an analogue of his result for complex polynomials.

This talk will present the following Rolle's theorem for complex polynomials. If $p(z)$ is a complex polynomial of degree $n \geq 5$, satisfying $p(-i) = p(i)$, then there is at least one critical point of p in the union $D[-c; r] \cup D[c; r]$ of two closed disks with centres $-c, c$ and radius r, where

$$c = \cot(2\pi/n), \quad r = 1/\sin(2\pi/n).$$

If $n = 3$, then the closed disk $D[0; 1/\sqrt{3}]$ has this property; and if $n = 4$ then the union of the closed disks $D[-1/3; 2/3] \cup D[1/3; 2/3]$ has this property. In the last two cases, the domains are minimal, with respect to inclusion, having this property.

This theorem is stronger than any other known Rolle's Theorem for complex polynomials. In particular, it answers Tschakaloff's question for polynomials of degree 3 and 4.

This is a joint work with Blagovest Sendov from the Bulgarian Academy of Sciences.