Equivariant geometry and topology
Géométrie et topologie équivariantes

(Org: **Jeffrey Carlson** and/et **Elisheva Adina Gamse** (University of Toronto))

TONY BAHRI, Rider University
Topology and geometry of polyhedral products

Cartesian products have embedded within them certain natural subsets which are indexed by combinatorial information. The geometric characterization of these subspaces, known now as *polyhedral products*, has wide application in toric geometry and topology, combinatorics, geometric group theory, number theory, free groups, homotopy theory and arachnid mechanisms. The cohomology of these spaces will be described briefly from complementary algebraic and geometric perspectives.

CHRISTIN BIBBY, University of Western Ontario
Representation stability for the cohomology of arrangements associated to root systems

From a root system, one may consider the arrangement of reflecting hyperplanes, as well as its toric and elliptic analogues. The corresponding Weyl group acts on the complement of the arrangement and hence on its cohomology. We consider a sequence of linear, toric, or elliptic arrangements which arise from a family of root systems of type A, B, C, or D, and we study the rational cohomology as a sequence of Weyl group representations. Our techniques combine a Leray spectral sequence argument similar to that of Church in the type A case along with FI_W-module theory which Wilson developed and used in the linear case.

ANNA MARIE BOHMANN, Vanderbilt University
Constructing equivariant cohomology theories

Equivariant cohomology theories that are cohomology theories incorporate a group action on spaces. These types of cohomology theories are increasingly important in algebraic topology but can be difficult to understand or construct. In recent work, Angelica Osorno and I have developed a construction for building them out of purely algebraic data based on symmetric monoidal categories. Our method is philosophically similar to classical work of Segal on building nonequivariant cohomology theories. In this talk I will discuss this work, and as well as an extension to the more general world of Waldhausen categories. Our new construction is more flexible and is designed to be suitable for equivariant algebraic K-theory constructions.

PETER CROOKS, University of Toronto
Equivariant projective compactifications of semisimple orbits

Semisimple adjoint orbits are affine varieties of interest to both algebraic and symplectic geometers. In particular, these orbits have been studied in the context of mirror symmetry, for which some understanding of their projective compactifications has been useful.

I will begin with an overview of complex adjoint orbits and their salient features, subsequently specializing to the semisimple case. Having done this, I will describe some joint work with Steven Rayan on constructing and understanding projective compactifications of semisimple orbits.

LAURA ESCOBAR, The Fields Institute and University of Illinois at Urbana Champaign
Torus orbits inside matrix Schubert and brick varieties

Given a projective algebraic variety X with an action of a torus T consider the closure of the torus orbits a point in the variety. If we start with a general point then its orbit closure is the toric variety of the moment polytope of the variety. This polytope allows us to verify if X is a toric variety with respect to T. We present this framework in the context of matrix Schubert varieties and brick varieties.

MATTHIAS FRANZ, University of Western Ontario
A quotient criterion for syzygies in equivariant cohomology

We start by reviewing the theory of syzygies in equivariant cohomology for actions of a compact connected Lie group G as initiated by Allday, Puppe and the speaker. Among other things, it permits to characterize the G-spaces for which a GKM-type description of equivariant cohomology holds. We will highlight the role played by a suitably defined equivariant homology, and we will discuss big polygon spaces as non-trivial examples.

We finally present a criterion that permits to read off the syzygy order of the equivariant cohomology of a manifold with a torus action from the orbit space. This criterion unifies and generalizes several results about the freeness and torsion-freeness of equivariant cohomology for various classes of such manifolds.

CHEN HE, Northeastern University
GKM descriptions of the equivariant cohomology rings of homogeneous spaces

For a homogeneous space G/H where G and H are of the same rank and both compact connected, Guillemin, Holm and Zara gave the GKM description of its equivariant cohomology ring. In this talk, we will use generalized GKM theories in both even and odd dimensions to consider the case where G and H have rank difference at most 1 and not necessarily connected, including the interesting examples of certain class of real or oriented flag manifolds.

PO HU, Wayne State University
Derived Representation Theory

I will talk about the construction of a theory of Lie algebras and their representations,making use of M. Ching's Lie operad and Elmendorf-Mandell's infinite loop space machinery. In particular, I will discuss the construction of spectra version of concepts from representation theory, such as Verma modules and Harish-Chandra pairs, as well as the application of categorifying sl_k link invariants in the category of spectra. (joint work with I. Kriz and P. Somberg)

LISA JEFFREY, University of Toronto
The genus 2 moduli space

(Joint with Nan-Kuo Ho, Khoa Dang Nguyen and Eugene Xia)

The moduli space of gauge equivalence classes of flat connections on a 2-manifold of genus 2 was proved to be isomorphic to complex projective 3-space by Narasimhan and Ramanan in 1969. Their proof used algebraic geometry. We use the Hamiltonian flows of Goldman (which were shown by Jeffrey and Weitsman to give moment maps for Hamiltonian circle actions on an open dense set) to give an identification between an open dense set of this moduli space and an open dense set of complex projective space.

YAEL KARSHON, University of Toronto
Maximal torus actions on complex manifolds with fixed points

I will report on joint work with Hiroaki Ishida that appeared in Math. Research Letters in 2012. We show that if a holomorphic n dimensional compact torus action on a compact connected complex manifold of complex dimension n has a fixed point then the manifold is equivariantly biholomorphic to a smooth toric variety.

IGOR KRIZ, University of Michigan
Computations of ordinary equivariant cohomology for powers of cyclic groups of prime order

I will talk about my joint work with John Holler on computing the $RO(G)$-graded coefficients of ordinary equivariant cohomology with coefficients $\mathbb{Z}/p$ where G is a power of $\mathbb{Z}/p$. I will elaborate on the key step of computing the coefficients of the corresponding "geometric fixed point" spectrum and certain related spectra. By recent results of Sophie Kriz, these computations are closely related to the theory of hyperplane arrangements.

SOPHIE KRIZ, Summers-Knoll school
Equivariant cohomology and the super reciprocal plane of a hyperplane arrangement

Motivated by computations of equivariant cohomology, I will talk about a certain object of algebraic geometry, namely a graded-commutative version of the reciprocal plane of a hyperplane arrangement. I will also discuss a superscheme version of a related toric compactification.

JEREMY LANE, University of Toronto
Symplectic invariants of focus-focus singularities and bifurcations of integrable systems

[joint in progress with Daniele Sepe] Since Duistermaat (1980) it has been known that focus-focus singularities obstruct an integrable system from being toric; these singularities introduce a global topological obstruction known as monodromy. More recently, Vu Ngoc (2002) showed that the presence of focus-focus singularities in integrable systems introduces more than just topological monodromy: for each focus-focus fibre there is an invariant that classifies the germ of the Lagrangian foliation up to symplectomorphism.

At the same time, one can often deform an integrable system so that the focus-focus singularity bifurcates into an elliptic singularity, and elliptic singularities have no such invariants. The obvious question is therefore: what happens to Vu Ngoc's symplectic invariants as the system bifurcates?

As a first step towards answering this question, we have undertaken the computation of symplectic invariants in the Hamiltonian-Hopf bifurcation. The Hamiltonian-Hopf bifurcation is a family of integrable systems on $\mathbb{R}^4$ that interpolates between the harmonic oscillator and the particle in a champagne bottle, introduced by Bates, which was one of the first systems known to possess focus-focus singularities. In this talk I hope to share some preliminary results and ideas they inspire.

YIANNIS LOIZIDES, University of Toronto
[Q,R]=0 and Verlinde Series

Let G be a compact, connected, simply connected Lie group, and let LG denote the loop group. There is a one-one correspondence between proper Hamiltonian LG-spaces and compact quasi-Hamiltonian G-spaces. We prove a 'norm-square localization' formula for the quantization of a quasi-Hamiltonian G-space, with terms indexed by the components of the critical set of the norm-square of the moment map of the corresponding LG-space. An important application is to give a new proof of a quantization-commutes-with-reduction theorem. This is joint work with E. Meinrenken.

LIVIU MARE, University of Regina
Equivariant cohomology of cohomogeneity one actions

I will mainly speak about actions of compact Lie groups on closed manifolds whose cohomogeneity (i.e., codimension of principal orbits) is equal to 1. The first result I will mention says that the equivariant cohomology module of any such action is Cohen-Macaulay. A special attention will be paid to the actions which are equivariantly formal. Finally, I will discuss the difficulties that arise in the study of actions whose cohomogeneity is greater than 1. This is joint work with Oliver Goertsches (University of Marburg, Germany).

ECKHARD MEINRENKEN, University of Toronto
Spinor bundles for Hamiltonian loop group spaces

Let G be a compact Lie group, and N a Hamiltonian LG space, with proper moment map.Associated to these data is a finite dimensional quasi-hamiltonian G-space M with G-valued moment map. We will give a geometric construction of a canonical "twisted spin-c structure" on M. This is based on joint work with Yiannis Loizides and Yanli Song.

LEONARDO MIHALCEA, Virginia Tech
Chern-Schwartz-MacPherson classes for Schubert cells and characteristic cycles

The Chern-Schwartz-MacPherson (CSM) class of a variety X is a class in the homology of X. In the case when X is a compact manifold, it coincides with the total Chern class of the tangent bundle of X. Its existence was conjectured by Deligne and Grothendieck, and it was first constructed by MacPherson. One can associate a CSM class to any constructible subset of X, and I will explain how one calculates this class for a Schubert cell in a (generalized) flag manifold G/P. It turns out that these classes are closely related to characteristic cycles of Verma D-modules on the cotangent bundle of G/P, and to Maulik and Okounkov's stable envelopes. This is based on joint work with P. Aluffi, and on ongoing joint work with P. Aluffi, J. Schürmann and C. Su.

DOUG RAVENEL, University ofRochster
The slice filtration revisited

The slice spectral sequence is the main computational device used to prove the Kervaire invariant theorem. It is based on a certain filtration of the category of G-spectra for a finite group G which generalizes the filtration by connectivity in the nonequivariant case. Using geometric fixed points, we give a new definition of it that is easier to work with than the original one.

REYER SJAMAAR, Cornell University
Convexity properties of presymplectic Hamiltonian actions

I will present a generalization of the convexity theorem of Atiyah, Guillemin-Sternberg and Kirwan, to Hamiltonian Lie group actions on presymplectic manifolds. The result implies convexity theorems for actions on contact manifolds, cosymplectic manifolds, as well as certain symplectic "generalized manifolds". This is joint work with Yi Lin.

MENTOR STAFA, Indiana University Purdue University Indianapolis (IUPUI)
Spaces of commuting elements in Lie groups

Spaces of group homomorphisms $Hom(\pi, G)$ from a discrete group to a Lie group have been studied in various contexts. We study the space of pairwise commuting n-tuples, i.e. π is free abelian, in a compact and connected Lie group G, from the topological viewpoint. We will describe a way to stabilize spaces of homomorphisms by introducing an infinite dimensional topological space, reminiscent of a Stiefel variety, that assembles the spaces of commuting tuples into a single space. Hilbert-Poincare series will be also described, in addition to other properties.

DONALD STANLEY, University of Regina
Connected sums of quasi-toric manifolds manifolds.

Quasi-toric manifolds are topological versions of toric manifolds. They have even dimension and have an action of a torus of half the dimension. If we start with two such manifolds of the same dimension we can glue the two manifolds together along orbits of this action. We describe the rational homotopy type of the resulting connected sum. This is joint work with Soumen Sarkar.

MARC STEPHAN, University of British Columbia
On free actions by elementary abelian p-groups

Carlsson conjectured that if a finite complex admits a free action by an elementary abelian p-group of rank n, then the sum of its mod-p Betti numbers is at least 2^n. For the prime $p = 2$, he reduced the conjecture to an algebraic problem which he solved for low n. In this talk, I will report on joint work in progress with Jeremiah Heller with the goal of extending Carlsson's methods to all primes. The crucial ingredient is a new notion of Koszul p-complexes.