ALEXANDER CHERNYAVSKY, McMaster University

Long-Time Stability of Standing Waves in Hamiltonian $\mathcal{PT}$-symmetric Chains of Coupled Pendula

I consider the Hamiltonian version of a $\mathcal{PT}$-symmetric lattice that describes dynamics of coupled pendula perturbed by a periodic resonant movement of their bases. Newton's equations of motion are reduced asymptotically to the $\mathcal{PT}$-symmetric discrete nonlinear Schrödinger equation. In the limit of weak coupling between the pendula, existence of periodic synchronized oscillations supported near one pair of coupled pendula follows by standard bifurcation analysis. If the gain-damping parameter that corresponds to the periodic resonance force is sufficiently small, spectral stability of such synchronized oscillations can be proved within the same limit. As the main contribution, I prove the nonlinear long-time stability of the synchronized oscillations by using the Lyapunov method. The periodic movement of coupled pendula is a saddle point of a constrained Hamiltonian function, which exists between the continuous bands of positive and negative energy. Nevertheless, I construct the approximate Lyapunov function and use it for the proof of nonlinear long-time stability of the synchronized oscillations of the coupled pendula.