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On similarity for completely bounded representations of Fourier algebras

Let G be a locally compact group. Dixmier's unitarization theorem for bounded continuous group representations may be restated as follows: if G is amenable, then every bounded representation for the group algebra on a Hilbert space, $\pi : L^1(G) \rightarrow \mathcal{B}(\mathcal{H})$, admits an invertible S in $\mathcal{B}(\mathcal{H})$ for which

$$S\pi(\cdot)S^{-1} \text{ is a } * \text{-representation, and } \|S\| \|S^{-1}\| \leq \|\pi\|^2. \quad (\dagger)$$

In the '90s, Pisier showed that (\dagger) implies amenability of G . The Fourier algebra $A(G)$ is the dual object to $L^1(G)$ in a manner which generalizes Pontryagin duality. It is a commutative self-adjoint Banach algebra of functions on G which is the predual of the von Neuman algebra generated by the left regular representation of G . As such, the operator space structure on $A(G)$ is generally non-trivial. However, every $*$ -representation of $A(G)$ factors through the commutative C^* -algebra of continuous functions vanishing at infinity $\mathcal{C}_0(G)$, and hence is completely bounded. Due to the considerations around the duality of $A(G)$ with $L^1(G)$, we suspect that for any completely bounded representation $\pi : A(G) \rightarrow \mathcal{B}(\mathcal{H})$ that there is an S in $\mathcal{B}(\mathcal{H})$ for which an analogue of (\dagger) holds. H.H. Lee (Seoul) and E. Samei (Saskatchewan) and I have found a proof for this result for a wide class of groups which includes amenable groups and small-invariant neighbourhood (hence discrete) groups.