YUVAL FILMUS, Technion – Israel Institute of Technology
Triangle-intersecting families of graphs

How many graphs can a collection of subgraphs of K_n consist of, if the intersection of any two contains a triangle? Simonovits and Sós asked this question in 1976, in the context of Erdös–Ko–Rado theory. They conjectured that such a collection can contain at most 1/8 of the graphs. This bound is attained by a "triangle-junta", consisting of all supergraphs of a fixed triangle.

It is trivial to see that such a family contains at most 1/2 of the graphs. Chung, Frankl, Graham, and Shearer improved this bound to 1/4 in 1986, using Shearer's entropy lemma. This is where matters stood until Ellis, Friedgut, and myself proved the optimal upper bound 1/8 in 2010.

Our proof uses a spectral technique known variously as Hoffman's bound, the Lovász theta function, and Friedgut's method. The technique involves cooking up a matrix whose rows and columns are indexed by subgraphs of K_n, entries corresponding to non-triangle-intersecting pairs of subgraphs are 0, the maximal eigenvalue is 1, and the minimal eigenvalue is –1/7.

Our main tool is the cut distribution of a graph, which is the distribution of the number of edges cut by a random partition of the vertex set. We find a linear combination of cut probabilities whose value always ranges between 1 and –1/7, and use it to construct the matrix mentioned above.