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Analysis of Boolean functions on exotic domains

Analysis of Boolean functions is the study of $0/1$-valued functions using spectral and analytic techniques. Lying at the interface of combinatorics, probability theory, functional analysis and theoretical computer science, it has been applied to random graph theory, percolation theory, coding theory, social choice theory, extremal combinatorics, and theoretical computer science.

Traditionally the functions being considered are on the Boolean cube $\{0,1\}^n$, or more rarely on other product domains, usually finite. We explore functions on more exotic domains such as finite groups and association schemes, concentrating on two examples: the symmetric group and the Johnson association scheme (the "slice"), which consists of all vertices in the Boolean cube with a specified weight.

We will survey a few classical results in analysis of Boolean functions on the Boolean cube and their generalizations to more exotic domains. On the way, we will explore questions such as: Which functions on the symmetric group are "dictatorships" (depend on one "coordinate") or "juntas" (depend on a few "coordinates")? Is there a "Fourier expansion" for functions on the slice? Is the middle slice a "representative" section of the Boolean cube?

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