Analysis on Singular Manifolds
Analyse sur des variétés singulières
(Org: **Alexey Kokotov** (Concordia) and/et **Frédéric Rochon** (UQAM))

YANNICK BONTHONNEAU, CIRGET - CRM

Continuous and discrete Weyl laws for negatively curved cusp manifolds

The purpose of my talk will be to give a new refinement in the spectral theory of the Laplace operator on manifolds with exact hyperbolic cusps, when the curvature is negative. I will give estimates on some spectral counting functions.

The usual counting function for the point spectrum $N(T)$ is not the relevant quantity as the Laplace operator has continuous spectrum. Instead, we have to consider $N(T)+S(T)$, where $S(T)$ is the phase shift in the appropriate scattering problem. First, when the curvature is negative, we will see that

$$N(T)+S(T)=\frac{\mathrm{vol}(B^*M)}{(2\pi)^n}T^n+aT\log T+bT^{n-1}+\mathcal{O}\left(\frac{T^{n-1}}{\log T}\right)$$

with some constants a and b.

The function $z\mapsto \exp iS(i((n-1)/2-z))$ has a meromorphic extension to $\mathbb{C}$, with its poles contained in $\Re z<(n-1)/2$. These poles can be seen as a discrete spectral set replacing the eigenvalues in the compact case, they are called resonances. I proved that for a generic set of metrics, most of these resonances are contained in a strip $\{\delta<\Re z<(n-1)/2\}$ at high frequency, where $\delta<(n-1)/2$. Under this genericity assumption, we will see that

$$\#\{s \text{ resonance} \mid \Re s>\delta,\ |\Im s|\leq T\}=AT^n+BT^{n-1}+CT\log T+\mathcal{O}\left(\frac{T^{n-1}}{\log T}\right).$$

DMITRY JAKOBSON, McGill

Gaussian measures on manifolds of metrics

This is joint work with Y. Canzani, B. Clarke, N. Kamran, L. Silberman and J. Taylor. We define Gaussian measures on manifolds of metrics with the fixed volume form. We next compute the moment generating function for the L2 (Ebin) distance to the reference metric. We also prove integrability results (with respect to our measures) for functionals such as Laplace eigenvalues, diameter, and volume entropy.

VICTOR KALVIN, Concordia University

Moduli spaces of meromorphic functions and determinant of Laplacian

The Hurwitz space is the moduli space of pairs (X,f), where X is a compact Riemann surface and f is a meromorphic function on X. We consider the Laplace operator on the flat non-compact singular Riemannian manifold $(X,|df|^2)$. We define a regularized relative determinant of the Laplace operator and obtain an explicit expression for the determinant in terms of the basic objects on the underlying Riemann surface (the prime form, theta-functions, the canonical meromorphic bidifferential) and the divisor of the meromorphic differential df. A surgery formula of the type of Burghelea-Friedlander-Kappeler (for the relative determinant of the Laplace operator on singular flat surfaces with conical and Euclidean ends) allows to close the conical/Euclidean ends and thus reduces the proof of explicit expression for the relative determinant to the proof of a similar expression for the zeta-regularized determinant of Laplace operator on the (compact) manifold with closed ends. The talk is based on a joint work with Alexey Kokotov and Luc Hillairet.

SPIRO KARIGIANNIS, University of Waterloo
Existence of G_2 conifolds: a progress report

Compact G_2 manifolds with isolated conical singularities, also called G_2 conifolds, are important objects in physics. They are expected to exist in abundance but no examples have yet been found. I will review past work of myself (on desingularization) and myself with Jason Lotay (on deformation theory) that gives strong mathematical evidence for their existence. Then I will discuss our current progress (with Lotay) on a possible construction of G_2 conifolds, by generalizing a new construction of myself and Dominic Joyce of smooth compact G_2 manifolds.

CHRIS KOTTKE, Northeastern University
Partial compactification and metric asymptotics of monopoles

I will describe a partial compactification of the moduli space, M_k, of $\mathrm{SU}(2)$ magnetic monopoles on $\mathbb{R}^3$, wherein monopoles of charge k decompose into widely separated 'monopole clusters' of lower charge going off to infinity at comparable rates. The hyperkahler metric on M_k has a complete asymptotic expansion, the leading terms of which generalize the asymptotic metric discovered by Bielawski, Gibbons and Manton in the case that the monopoles are all widely separated. This is joint work with M. Singer, and is part of a larger work in progress with R. Melrose and K. Fritzsch to fully compactify the M_k as manifolds with corners and compute their L^2 cohomology.

THOMAS KRAINER, Penn State Altoona
Boundary Value Problems for Elliptic Wedge Operators of First Order

Spaces with singularities of edge type are modeled by a smooth compact manifold with boundary, where the boundary is the total space of a locally trivial fibration. The relation with the singular space is given by collapsing the fibers to points. On such manifolds there is a natural class of incomplete Riemannian metrics (incomplete-edge metrics) that reflect the conic degeneration of the fibers at the boundary. Geometric operators associated with such metrics are examples for the broader class of wedge differential operators. In the case of the trivial boundary fibration (fiber is a point), incomplete-edge metrics are precisely the smooth metrics up to the boundary, and the class of wedge differential operators includes all regular differential operators with smooth coefficients up to the boundary.

In this talk, I will report about joint work with Gerardo Mendoza addressing the problem of well-posedness of elliptic equations for wedge operators of first order. Central to the investigation is the development of an appropriate notion of boundary condition associated with the singular locus (i.e. the edge). In the case of the trivial boundary fibration, our theory includes the classical theory of elliptic boundary problems for first-order operators as a special case.

GERARDO MENDOZA, Temple University
Spectral instability of selfadjoint extensions of symmetric elliptic cone operators

Let $A : C_c^\infty(\dot{M};E) \subset x^{-\nu/2}L_b^2(\dot{M};E) \to x^{-\nu/2}L_b^2(\dot{M};F)$ be an elliptic cone operator acting on sections of a vector bundle E over a smooth compact manifold M with boundary $\partial M = \{x = 0\}$. Suppose A is symmetric, bounded from below, and admits more than one selfadjoint extension. The family, $\mathfrak{SA}$, of domains of such extensions is a smooth compact real-analytic manifold. The spectrum of A with any domain $D \in \mathfrak{SA}$ is bounded below, but there exist domains D_0 which admit a neighborhood $U \subset \mathfrak{SA}$ for which the property $\forall \zeta \in \mathbb{R}\ \exists D \in U$ s.t. $\inf \operatorname{spec}(A_D) < \zeta$ holds. The set of such domains is a codimension 1 (real-)analytic variety in $\mathfrak{SA}$ which will be described explicitly.

ERIC WOOLGAR, University of Alberta
APEs

I will discuss asymptotically hyperbolic manifolds and sub-types, especially Asymptotically Poincaré-Einstein (APE) manifolds. These are manifolds whose metrics admit a Fefferman-Graham type expansion near conformal infinity, such that the first few leading terms are determined by the Einstein equations. APEs appear in physics, both as time slices of asymptotically Anti-de Sitter (AdS) spacetimes and as Wick rotations of static AdS spacetimes. Among the invariants that can (sometimes) be associated to APEs are mass and renormalized volume. For APEs that represent static AdS black holes in 4 dimensions, the renormalized volume and the mass of a static slice can be related. Remarkably, this relationship, which can be established purely on geometric grounds, is the Gibbs relation of black hole thermodynamics. A related result is that APEs which are time-symmetric slices of globally static AdS spacetimes cannot have positive mass. Examples are provided by Horowitz-Myers geons, which are complete APEs with scalar curvature $S = -n(n-1)$ and negative mass. They serve as time-symmetric slices of so-called AdS solitons, which are Einstein spacetimes with toroidal conformal infinity. The Horowitz-Myers "positive" energy conjecture proposes that Horowitz-Myers geons minimize the mass over all APEs with scalar curvature $S \geq -n(n-1)$ and the same toroidal conformal infinity. The conjecture is a major open problem in the field, but it is possible to show that if this conjecture is true then there is a rigidity theorem for Horowitz-Myers geons: the least-mass geon is the unique complete APE having that mass and obeying $S \geq -n(n-1)$.