Analytic Number Theory
Théorie analytique des nombres

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AMIR AKBARY-MAJDABADNO, University of Lethbridge

On invariants of elliptic curves on average

For an elliptic curve E defined over $\mathbb{Q}$ and a prime p of good reduction, it is known that the group of rational points $E_p(\mathbb{F}_p)$ of the reduction mod p of E over the finite field $\mathbb{F}_p$ is the product of at most two cyclic groups. Let $i_E(p)$ be the index of the largest cyclic subgroup of $E_p(\mathbb{F}_p)$. We describe a general theorem regarding certain functions of $i_E(p)$ on average over the family of all elliptic curves inside a box. We derive several results related to some invariants of elliptic curves as corollaries to this general theorem. This is a joint work with Adam Felix (KTH, Sweden).

KEVSER AKTAS, Queen's University

On Special Numbers

A natural number n is special if in its prime factorizatation $n = p_1^{\alpha_1}, \ldots, p_k^{\alpha_k}$ we have all α_i distinct. Let $V(x)$ be the number of special numbers $\leq x$. We will show that there is a constant $c > 1$ such that $V(x) \sim \dfrac{cx}{\log x}$. We will make some remarks on determining the error term at the end. This is a joint work with Prof. M. Ram Murty.

FARZAD ARYAN, University of Lethbridge

On the Quadratic Divisor Problem

Let $d(n) = \sum_{d|n} 1$ denote the divisor function. It counts the number of divisors of an integer. Consider the following shifted convolution sum

$$\sum_{an-m=h} d(n)d(m)f(an,m),$$

where f is a smooth function which is supported on $[x, 2x] \times [x, 2x]$ and oscillates mildly. In 1993, Duke, Friedlander, and Iwaniec proved that

$$\sum_{an-m=h} d(n)d(m)f(an,m) = \text{Main term}(x) + O(x^{0.75+\epsilon}).$$

Here, we improve the error term in the above formula to $O(x^{0.61})$, and conditionally, under the assumption of the Ramanujan-Petersson conjecture, to $O(x^{0.5+\epsilon})$. We will also present some new results on shifted convolution sums of functions coming from Fourier coefficients of modular forms.

SANDRO BETTIN, ICERM - University of Genova

On the average root number of 1-parameter families of elliptic curves

We discuss the behaviour of the average of the root number in 1-parameter families of elliptic curves. By the work of Helfgott, in a typical family the root number has average zero, but it can happen in special circumstances that the average is non-zero. We study these special cases (with particular attention to the case when the parameter varies among integers), and show that all rational numbers between in $[-1, 1]$ can be average root numbers.

ARUNABHA BISWAS, Queen's University

Asymptotic nature of higher Mahler Measure.

We consider the k-higher Mahler measure $m_k(P)$ of a Laurent polynomial P as the integral of $\log^k |P|$ over the complex unit circle. In this talk we present an explicit formula for the value of $|m_k(P)| \ / \ k!$ as $k \to \infty$. We also present the rate of convergence of the sequence $\{m_k(P)\}_{k\geq 1}$ for the representative special case $P(z) = z + r$ with $|r| = 1$.

CHANTAL DAVID, Concordia University
One-level density in one-parameter families of elliptic curves with non-zero average root number

(joint work with Sandro Bettin and Christophe Delaunay)

We present in this talk a (conjectural) formula for the one-level density of general one-parameter families of elliptic curves, in term of n, the rank of E over $Q(t)$ and the average root number W_E over the family. In the general case, W_E is zero, and the one-level density is given by orthogonal symmetries as predicted by the conjectures of Katz and Sarnak. In the exceptional cases where $W_E \neq 0$, we find that the statistics are given by a weighted sum of even orthogonal and odd orthogonal symmetries. The most dramatic and counter-intuitive cases occur when $W_E = \pm 1$. In that case, the one-level density exhibits even orthogonal symmetries when $(-1)^n W_E = 1$ and odd orthogonal symmetries when $(-1)^n W_E = -1$, and there is a shift of the symmetries (between orthogonal odd and orthogonal even) when n is odd.

DIMITRI DIAS, Université de Montréal
Apollonian packings

Apollonian packings date back to ancient Greece and, from a number theoretical point of view, are very attractive objects. In this talk, we will discuss the properties of the set of curvatures (the inverses of the radii) of such packings and some of their generalizations. Under the right conditions, these curvatures are integers. These integers can be studied and counted, and some of these results will be exposed here.

KARL DILCHER, Dalhousie University
Generalized Fermat numbers and congruences for Gauss factorials

We define a Gauss factorial $N_n!$ to be the product of all positive integers up to N that are relatively prime to $n \in \mathbb{N}$. We consider the Gauss factorials $\lfloor \frac{n-1}{M} \rfloor_n!$ for $M = 3$ and 6, where the case of n having exactly one prime factor of the form $p \equiv 1 \pmod 6$ is of particular interest. A fundamental role is played by primes with the property that the order of $\frac{p-1}{3}!$ modulo p is a power of 2 or 3 times a power of 2; we call them Jacobi primes. Our main results are characterizations of those $n \equiv \pm 1 \pmod M$ of the above form that satisfy $\lfloor \frac{n-1}{M} \rfloor_n! \equiv 1 \pmod n$, $M = 3$ or 6, in terms of Jacobi primes and certain prime factors of generalized Fermat numbers. (Joint work with John Cosgrave).

TRISTAN FREIBERG, University of Waterloo
Prime gaps: small, medium and large

We discuss the distribution of primes in short intervals from the point of view of the Cramér random model. We state two related conjectures and present recent work motivated by them. This work involves an incorporation of the Erdős–Rankin construction (large prime gaps) into Maynard's sieve (small prime gaps).

SUMIT GIRI, Centre de Recherches Mathématiques
On correlation of certain multiplicative functions

In this talk, we consider the correlation function of the form $\frac{1}{x}\sum_{n\leq x} F(n)G(n-h)$, were F and G are close to 1 on primes. Also, replacing F by μ^2, we compute an average of shifted Euler totient function $\phi(n-h)$ over square-free integers n and in general sums of the type $\sum_{n\leq x} \mu^2(n)G(n-h)$, where G is close to 1 on primes. This is joint work with R. Balasubramanian and Priyamvad Srivastav.

KEVIN HARE, University of Waterloo
Some properties of even moments of uniform random walks

We build upon previous work on the densities of uniform random walks in higher dimensions, exploring some properties of the even moments of these densities and extending a result about their modularity.

HABIBA KADIRI, University of Lethbridge
New bounds for $\psi(x;q,a)$

The prime number theorem in arithmetic progressions establishes that, for a and q fixed coprime integers, then $\psi(x;q,a)$ is asymptotic to $\frac{x}{\phi(q)}$ when x is large. We discuss new explicit bounds for the error term which provide an extension and improvement over the work of Ramaré and Rumely. Such results depend on new results about the zeros of the Dirichlet L-functions of respectively Platt and Kadiri. In addition our method makes use of smooth weights. This is joint work with Allysa Lumley.

YOUNESS LAMZOURI, York University
Large values of $L(1,\chi)$ and applications to character sums and class numbers of real quadratic fields

We investigate large values of $L(1,\chi)$ where χ varies over certain special families of Dirichlet characters. More specifically, we consider the family of primitive characters of a fixed order k, and the family of quadratic characters χ_d corresponding to fundamental discriminants d of the form $4n^2+1$. For each family we produce large values of $L(1,\chi)$ which are believed to be best possible. We then apply our results to exhibit extreme values of even order character sums, and of class numbers of real quadratic fields, which are likely optimal.

STEVE LESTER, KTH Royal Institute of Technology
Small scale equidistribution of eigenfunctions on the torus

I will describe some recent results on the distribution of the L^2-mass of eigenfunctions of the Laplacian on the torus $\mathbb{T}^d/2\pi\mathbb{R}^d$. A special case of a result of Marklof and Rudnick implies that the L^2-mass of almost all such eigenfunctions equidistributes with respect to Lebesgue measure for $d=2$. I will discuss results on the scales at which the L^2-mass equidistributes, as well as mention some limitations on equidistribution, and relate these questions to arithmetic problems such as representing integers as sums of squares and the distribution of lattice points. This is joint work with Zeév Rudnick.

YU RU LIU, University of Waterloo
Vinogradov's mean value theorem in function fields

Vinogradov's mean value theorem is about integer solutions of the system of equations $x_1^j+\cdots+x_s^j=y_1^j+\cdots+y_s^j$ $(1\le j\le k)$. In this talk, we will give a quick survey about the recent progress on this problem, based on Wooley's efficient congruencing method. Then we will talk about its generalizations to function fields. It is joint work with W. Kuo, T. Wooley, and X. Zhao.

ALLYSA LUMLEY, York University
A Zero Density Result for the Riemann Zeta Function

Let $N(\sigma,T)$ denote the number of non-trivial zeros of the Riemann zeta function with real part greater than σ and imaginary part between 0 and T. We provide explicit upper bounds for $N(\sigma,T)$ commonly referred to as a zero density result. In 1940, Ingham showed the following asymptotic result

$$N(\sigma,T)=O(T^{\frac{3(1-\sigma)}{2-\sigma}}\log^5 T).$$

Ramaré recently proved an explicit version of this estimate:

$$N(\sigma, T) \leq 4.9(3T)^{\frac{8}{3}(1-\sigma)} \log^{5-2\sigma}(T) + 51.5 \log^2 T,$$

for $\sigma \geq 0.52$ and $T \geq 2000$. We discuss a generalization of the method used in these two results which yields an explicit bound of a similar shape while also improving the constants. Furthermore, we present the effect of these improvements on explicit estimates for the prime counting function $\psi(x)$. This is joint work with Habiba Kadiri and Nathan Ng.

PATRICK MEISNER, Concordia University
Distribution of Points on Curves over Finite Fields

The distribution of the number points on curves over finite fields with a fixed Galois group has been a topic of much research recently. It began with Kurlberg and Rudnick determining the distribution of the number of points of hyper-elliptic curves. Hyper-elliptic curves are in one-to-one correspondence with Galois extensions of $\mathbb{F}_q(X)$ with Galois group $\mathbb{Z}/2\mathbb{Z}$. Bucur, David, Feigon and Lalin extended this result to smooth project curves that are in one-to-one correspondence with Galois extensions of $\mathbb{F}_q(X)$ with Galois group $\mathbb{Z}/p\mathbb{Z}$, where p is a prime such that $q \equiv 1 \mod p$. Recently Lorenzo, Milione and Meleleo determined the case for Galois group $(\mathbb{Z}/2\mathbb{Z})^n$. This talk will focus on the case where the Galois group is cyclic.

RAM MURTY, Queen's University
A disjunction theorem in sieve theory

Combining the higher rank sieve theory with the generalized Elliott-Halberstam conjecture, we prove that either the Artin primitive root conjecture or the Lang-Trotter conjecture for primitive points on CM elliptic curves is true. This is joint work with Akshaa Vatwani.

MAKSYM RADZIWILL, Rutgers University
Multiplicative functions in short intervals and applications

I will discuss joint work with Kaisa Matomaki on multiplicative functions in short intervals and in particular focus on applications to questions related to Chowla's conjecture (on correlations between consecutive values of the Mobius function). The applications are joint work with Kaisa Matomaki and Terence Tao.

MIKE RUBINSTEIN, University of Waterloo
Fun with integrals

We analyze a couple of multi-dimensional integrals that arise in the asymyptotics of elliptic aliquot cycles, and in the moments of the average k-th divisor function over short intervals.

JESSE THORNER, Emory University
Effective log-free zero density estimates for automorphic L-functions and the Sato-Tate conjecture

We prove two log-free zero density estimates for Rankin-Selberg L-functions over number fields with uniformity in their dependence on the analytic conductor. We consider applications of these estimates to automorphic variants of Hoheisel's short interval prime number theorem and Linnik's bound on the least prime in an arithmetic progression. We focus on applications in the context of the Sato-Tate conjecture for non-CM elliptic curves, where the uniformity is crucial.

This is joint work with Robert Lemke Oliver (Stanford University).

AKSHAA VATWANI, Queen's University
A higher rank Selberg sieve and applications

We present a general higher rank Selberg sieve and apply it to various questions in number theory. In particular, we improve upon a result of Heath-Brown on almost prime k-tuples. This is joint work with Professor Ram Murty.

CHESTER WEATHERBY, Wilfrid Laurier

Explicit evaluation of sums using derivative polynomials

A derivative polynomial refers to function, $f(x)$, whose derivative is a polynomial of the original function. As an example, $f(x) = e^x$ has derivative polynomial $P(x) = x$ since $f'(x) = P(f(x))$. When a function satisfies a derivative polynomial, all higher order derivatives also satisfy polynomial expressions. We will show that in some cases, derivative polynomials are useful in explicit evaluation of infinite sums, and in particular show a generalization of Euler's Theorem for evaluation of $\zeta(2k)$.

ASIF ZAMAN, University of Toronto

Distribution of zeros of Hecke L-functions and the least prime ideal

We discuss explicit results on the distribution of zeros of Hecke L-functions along with how it relates to bounding the least norm of a prime ideal in the Chebotarev Density Theorem. In particular, we exhibit quantitative versions of a "log-free" zero density estimate and a zero repulsion phenomenon for Hecke L-functions.

This is joint work with Jesse Thorner.