Algebraic Number Theory
Théorie algébrique des nombres
(Org: **Antonio Lei** (Laval))

DANIEL BARRERA, CRM
Overconvergent Eichler-Shimura isomorphisms for Shimura curves

We will discuss the p-adic variation of the Eichler-Shimura isomorphism in the context of Shimura curves over the rational numbers. In particular, we describe the finite slope part of the space of overconvergent modular symbols in terms of the finite slope part of the space of overconvergent modular forms. This is joint work with Shan Gao.

HUGO CHAPDELAINE, Laval University
Kronecker limit formulas for a class of GL_2-real analytic Eisenstein series

In order to motivate the main result of this talk, we will start by introducing a class of partial zeta functions associated to a totally real field K. The general term of these partial zeta functions are twisted simultaneously by a finite additive character ψ of K and a sign character ω of $K^\times$. Let us denote such a zeta function by $\zeta(s;\omega,\psi)$ where s varies over the complex plane. When ω is chosen to be $\omega_1 := sign(N_{K/\mathbb{Q}})$ and ψ is chosen to be the trivial additive character ψ_0, then the special value at $s=1$, up to some explicit fudge factor, is a rational number which corresponds to the signature defect of a cusp manifold included in a certain Hilbert modular variety associated to K. If one replaces the sign character ω_1 by another sign character ω, then, so far, no differential geometric interpretation has been provided for the special value $\zeta(1;\omega,\psi_0)$. In an attempt to study, from a differential geometric point of view, the value $\zeta(1;\omega,\psi_0)$, we will present a Kronecker limit formula for a class of Eisenstein series $E(z,s;\omega,\psi)$ which "interpolates" the partial zeta functions $\zeta(s;\omega,\psi)$. The interpolation here is in the sense that the constant term of the Fourier series expansion of $z\mapsto E(z,s;\omega,\psi)$ is of the form $\varphi_1(s)\zeta(2s;\omega,\psi)+\varphi_2(s)\zeta(2s-1;\omega,\psi)$ for some explicit functions $\varphi_1(s)$ and $\varphi_2(s)$. Here the parameter z varies over the corresponding Hilbert modular variety.

SUNGMUN CHO, University of Toronto
Rationality of the local density function

Local densities are local factors of the Siegel mass formula for an integral quadratic form over a number field. It is essential to compute all local densities for every prime in order to classify integral quadratic forms.

In this talk, I will explain that the local density function with respect to the size of a residue field, when we vary a base complete local ring to its finite unramified extension, is a rational function (i.e. polynomial divided by another polynomial).

DANIEL DISEGNI, McGill
On the p-adic Birch and Swinnerton-Dyer conjecture for elliptic curves over number fields

I will propose an analogue of the BSD conjecture in several p-adic variables, and discuss old and new theoretical evidence towards it.

LUIS GARCIA, University of Toronto
Theta lifts and currents on Shimura varieties

We will discuss a regularized theta lift for the dual pair $(Sp_4, O(V))$. This lift takes values in a space of currents that contains the regulators from certain higher Chow groups. We will state a formula relating the values of the currents that arise as theta lifts with a special value of a standard Sp_4 L-function.

EYAL GOREN, McGill University
Unitary Shimura varieties in positive characteristic

I will report on new results concerning the geometry of unitary Shimura varieties in positive characteristic. In particular, on the construction of a theta operator and the canonical subgroup. This is joint work with Ehud De Shalit (Hebrew University).

FLORIAN HERZIG, University of Toronto
On de Rham lifts of local mod p Galois representations

It is an open problem to show that a given n-dimensional mod p local Galois representation ρ has a lift to a p-adic Galois representation. One even hopes that a de Rham lift exists. We discuss results concerning the existence of de Rham lifts of ρ with prescribed properties, assuming that ρ admits a "nice" lift to start with. Our arguments combine local and global methods. This is joint work with T. Gee, T. Liu, and D. Savitt.

PATRICK INGRAM, Colorado State University (on leave)
Ramification in iterated extensions of number fields

Cyclotomic extensions, and torsion extensions for elliptic curves, give examples of infinite towers of number fields ramified only above a finite set of primes. Aitken, Cullinan, Hajir, and Maire showed that the splitting fields of iterates of post-critically finite rational functions of one variable have the same property, which is in some sense a direct generalization of the aforementioned cases. We will survey these results, and related questions in Galois theory. We will also present a generalization of, and a converse to, their theorem.

ADRIAN IOVITA, Concordia University
p-Adic Families of de Rham cohomology classes

Together with Fabrizio Andreatta we have constructed p-adic families, parameterized by the weights, of Banach modular sheaves with filtrations, connections and Frobenii which interpolate symmetric powers of the relative de Rham cohomology of the universal elliptic curve over a relevant neighborhood of the ordinary locus in a modular curve of level prime to p. This construction has as application an overconvergent crystalline Eichler-Shimura isomorphism.

DANIEL KRIZ, Princeton University
Congruences of p-adic L-functions and applications to algebraic cycles

I will discuss my result establishing a congruence between the Bertolini-Darmon-Prasanna anticyclotomic p-adic L-function attached to a newform f with reducible residual p-adic Galois representation and the Katz p-adic L-function. From this, there follows a congruence between p-adic Abel-Jacobi images of certain generalized Heegner cycles and products of certain Bernoulli numbers and Euler factors. As an application, one can show that when a semistable elliptic curve $E/\mathbb{Q}$ has reducible mod 3 Galois representation, ranks 0 and 1 each occur with a positive proportion in the quadratic twist family of E, and furthermore one can give explicit families of twists with these ranks. If time permits, I will also discuss recent work with Chao Li pertaining to relationships between ranks within quadratic twist families of elliptic curves.

DANIEL LE, University of Toronto
The weight part of Serre's conjecture for U(3)

Let F be a CM field in which p is unramified. For Galois representation $\overline{\rho} : G_F \to \mathrm{GL}_3(\overline{\mathbb{F}}_p)$ arising from the cohomology of a $U(3)$ arithmetic manifold which is tamely ramified and sufficiently generic at p, we describe the weights of $U(3)$ algebraic modular forms which give rise to $\overline{\rho}$ in terms of the restriction of $\overline{\rho}$ to the inertia groups at p. This confirms the $U(3)$ analogues of conjectures of Herzig and Emerton-Gee-Herzig-Savitt, which generalize Serre's conjectures in the case of classical

modular forms. We prove this by proving the first automorphy lifting results in semisimple rank greater than one outside the Fontaine-Laffaille range. This is joint work with Bao Viet Le Hung, Brandon Levin, and Stefano Morra.

CLAUDE LEVESQUE, U. Laval
Families of Thue-Mahler equations with only trivial solutions

This is joint work with Michel Waldschmidt. For each $n \geq 3$, we exhibit new families of Thue-Mahler equations having only trivial solutions. Furhermore, we produce an effective upper bound for the number of these solutions.

CHRISTIAN MAIRE, Université de Franche-Comté (Besançon)
On the linear growth of the p-rank of the Class Group: the pro-p groups point of view.

This is a joint work with Farhid Hajir. Genus theory may produce some situations where the p-rank of the p-class group grows linearly with the degree. In this talk,thanks to an idea of Ershov and a result of Schmidt, we explain this phenomena by using the description by generators and relations of a certain pro-p Galois group.

GUILLERMO MANTILLA-SOLER, Universidad de los Andes
Weak Arithmetic equivalence

Inspired by the invariant of a number field given by its zeta function we define the notion of weak arithmetic equivalence and show that for certain families of number fields, e.g., semistable(having fundamental discriminant), this equivalence determines the local root numbers of the number field. This is analogous to the fact that for semistable rational elliptic curves the local root numbers are determined by the bad part of the L-function of the curve.

GAUTIER POSINET, Université Laval
Functional equation for multi-signed Selmer groups

Let E be an elliptic curve defined over $\mathbb{Q}$ with supersingular reduction at a prime p. One problem to study the Iwasawa theory of E along the $\mathbb{Z}_p$-cyclotomic extension is the non co-torsioness of the Selmer group of E. This issue was resolved by Kobayashi by defining the plus/minus Selmer groups. The main conjecture that relate these Selmer groups to Pollack's plus/minus p-adic L-function on the analytic side, has recently been proved (Kato-Kobayashi-Wan). This implies a functional equational for the plus/minus Selmer groups, which had been independently proved by B.D. Kim. A. Lei and K. Büyükboduk have showed that the definition of plus/minus Selmer groups can be generalized to abelian varieties by . In this talk, we shall show that using techniques of Kim, these new Selmer groups satisfy a functional equation as in the case of elliptic curves.

SIDDARTH SANKARAN, McGill University
On two arithmetic theta lifts

I will report on joint work with Stephen Ehlen, in which we study certain families of Green functions attached to divisors on orthogonal and unitary symmetric spaces. Along the way, we will encounter a curious construction of a section of the Maass lowering operator for classical modular forms. Time permitting, I will also discuss applications to the arithmetic geometry of Shimura varieties.

FLORIAN SPRUNG, Princeton/IAS
The main conjecture for elliptic curves at supersingular primes

We sketch a proof of the Iwasawa main conjecture for elliptic curves at supersingular primes.

NAOMI TANABE, Dartmouth College
The First Moment of the Rankin-Selberg L-functions

In this talk, we discuss the twisted first moment of Rankin-Selberg convolutions at $s = 1/2$, associated to two Hilbert modular forms, as well as its applications; namely determining its underlying automorphic forms and non-vanishing of central L-values. This is an ongoing joint work with Alia Hamieh.

JACOB TSIMERMAN, University of Toronto
Bounding 2-torsion in Class group

(joint with Bhargava,Shankar,Taniguchi,Thorne, and Zhao) Zhang's conjecture asserts that for fixed positive integers m,n, the size of the m-torsion in the class group of a degree n number field is smaller than any power of the discriminant. In all but a handful of cases, the best known result towards this conjecture is the "convex" bound given by the Brauer-Siegel Theorem.

We make progress on this conjecture by giving a "subconvex" bound on the size of the 2-torsion of the class group of a number field in terms of its discriminant, for any value of n. The proof is surprisingly elementary, and we give several applications of this result stemming from the case of cubic fields, including improved bounds on the number of A4 fields, and on the number of integer points an elliptic curve can have.

Along the way, we prove a a surprising result on the shape of the lattice of the ring of integers of a number field. Namely, we show that such a lattice is very limited in how 'skew' it can be.

JAN VONK, McGill
Non-abelian Chabauty on higher genus curves

The recent work of Minhyong Kim marked the start of an exciting programme in search of effective versions of Diophantine finiteness theorems. We review some recent results of Dogra on rational points on certain classes of higher genus curves which go beyond the range of applicability of Chabauty's method, and discuss work in progress on effective versions of Faltings' theorem for curves whose Jacobians are simple and have real multiplication.

AMY WOODING, McGill
Stratifications of Unitary Shimura Varieties

I will describe some stratifications of unitary Shimura varieties and applications to their geometry in positive characteristic.