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Twisted highest weight modules and shellability

We will describe some aspects of (one variant of) a theory of twisted (thickened) highest weight modules for certain highest weight categories. This has as consequences general analogues (in some cases, stronger versions) of various well known phenomena in Lie theory. For instance, factorization of Shapovalov determinants in this setting is often a consequence of factorizations of Shapovalov matrices. It is possible to give a general construction of all highest weight categories admitting structure of this type, but difficult to understand the finer details of their combinatorics and to prove that natural highest weight categories arising in Lie theory (e.g. attached to Coxeter groups) admit such structure. However, extensive classes of examples are known in which the underlying combinatorics of twisted highest weight modules is especially simple and may be described in terms of classical notions of shellability; for example, shellable pure simplicial complexes may be characterized amongst pure simplicial complexes as exactly those for which certain associated highest weight representation categories admit such structure.