
Symplectic Geometry, Moment Maps and Morse Theory
Symplectic Géométrie symplectique, applications moment et théorie de Morse
(Org: Lisa Jeffrey (Toronto))

TOM BAIRD, Memorial University of Newfoundland
Moduli spaces of real vector bundles over a real curve

Given a Riemann surface X , we may associate a moduli space $M(X)$ of stable holomorphic vector bundles of some fixed rank and degree. If the rank and degree are coprime, then $M(X)$ is a compact Kähler manifold. Atiyah and Bott showed how $M(X)$ can be constructed as an infinite dimensional symplectic quotient, and used this to compute Betti numbers and other topological information.

Suppose now that X comes equipped with an anti-holomorphic involution τ . This induces an involution of $M(X)$ and the fixed point set, $M(X, \tau)$, is a real Lagrangian submanifold of $M(X)$. Biswas-Huisman-Hurtubise and Schaffhauser showed how $M(X, \tau)$ can be understood as a moduli space of real vector bundles over (X, τ) and can be constructed as an infinite dimensional "real Lagrangian quotient". In this talk, I will explain how the methods of Atiyah and Bott can be adapted to compute the Z_2 -Betti numbers of $M(X, \tau)$. I will also comment on how these $M(X, \tau)$ form a promising class of examples for Lagrangian Floer theory.

TATYANA BARRON, University of Western Ontario
Multisymplectic manifolds and quantization.

I will discuss how techniques of Berezin-Toeplitz quantization can be adapted to quantization on multisymplectic manifolds, including hyperkähler manifolds. I will briefly mention moment maps.

VIRGINIE CHARETTE, Université de Sherbrooke
Some symplectic geometry in the 3-d Einstein Universe

The 3-dimensional Einstein Universe is the conformal compactification of 3-dimensional affine Minkowski space. Interestingly, it can also be interpreted as the space of Lagrangian planes in a 4-dimensional symplectic vector space. Using a suitable dictionary, we will discuss how certain facts concerning the Einstein Universe can be nicely stated in this language.

PETER CROOKS, University of Toronto
Minimal nilpotent Hessenberg varieties

Hessenberg varieties constitute a diverse and interesting family of subvarieties of the flag variety. They are studied in a number of areas, including algebraic geometry, combinatorics, geometric representation theory, and equivariant topology. With respect to the last of these disciplines, there has been a great deal of interest in understanding the equivariant cohomology rings associated with appropriately defined torus actions on Hessenberg varieties.

Now, let G be a simply-connected simple group over \mathbb{C} , and fix a maximal torus and Borel subgroup, $T \subseteq B$, respectively. I will explain what it means for $X \subseteq G/B$ to be a minimal nilpotent Hessenberg variety. I will subsequently offer two descriptions of $H_T^*(X)$. The first arises from GKM theory, while the second description results from exhibiting $H_T^*(X)$ as a quotient of $H_T^*(G/B)$.

This represents joint work with Hiraku Abe

MAIA FRASER, University of Ottawa
Contact non-squeezing via contact homology

I will discuss non-squeezing in the contact manifold $\mathbb{R}^n \times S^1$ and a proof that “large-scale” prequantized balls, namely $B(R) \times S^1$ with $R > 1$, cannot be squeezed into themselves by a compactly supported contactomorphism. This statement and its proof by contact homology are in the same spirit as Eliashberg-Kim-Polterovich’s (2006) proof of non-squeezing for $R \in \mathbb{N}$. Chiu (2014) proved $B(R) \times S^1$, $R > 1$, cannot be squeezed into itself by a compactly supported contact isotopy; a similar result can be obtained by generating function arguments.

ADINA GAMSE, Northeastern University

Vanishing theorems in the cohomology ring of the moduli space of parabolic bundles

Let Σ be a compact connected oriented 2-manifold of genus g , and let p be a point on Σ . We define a space $S_g(t)$ consisting of certain irreducible representations of the fundamental group of $\Sigma \setminus p$, modulo conjugation by $SU(N)$. This space has interpretations in algebraic geometry, gauge theory and topological quantum field theory; in particular if Σ has a Kähler structure then $S_g(t)$ is the moduli space of parabolic vector bundles of rank N over Σ .

For $N = 2$, Weitsman considered a tautological line bundle on $S_g(t)$, and proved that the $(2g)^{th}$ power of its first Chern class vanishes, as conjectured by Newstead. In this talk I will present his proof and then outline my extension of his work to $SU(N)$ and to $SO(2n + 1)$.

EMMANUEL GIROUX, CNRS-CRM

Existence of Lefschetz fibrations on Stein and Weinstein domains

In this talk, I will present our joint work with John Pardon showing that every Stein or Weinstein domain may be realized (up to deformation) as a Lefschetz fibration over the disk. The proof is an application of Donaldson’s quantitative transversality techniques.

TYLER HOLDEN, University of Toronto

Non-abelian convexity of based loop groups

If K is a compact, connected, simply connected Lie group, its based loop group ΩK is endowed with a Hamiltonian $S^1 \times T$ action, where T is a maximal torus of K . Atiyah and Pressley examined the image of ΩK under the moment map μ , while Jeffrey and Mare examined the corresponding image of the real locus ΩK^τ for a compatible anti-symplectic involution τ . Both papers generalize well known results in finite dimensions, specifically the Atiyah-Guillemin-Sternberg theorem, and Duistermaat’s convexity theorem. In the spirit of Kirwan’s convexity theorem, I have generalized the two aforementioned results by demonstrating convexity of ΩK and its real locus ΩK^τ in the full non-abelian regime, resulting from the Hamiltonian $S^1 \times K$ action. In particular, this is done by appealing to the Bruhat decomposition of the algebraic (affine) Grassmannian, and appealing to the “highest weight polytope” results for Borel-invariant varieties of Guillemin and Sjamaar and Goldberg.

JACQUES HURTUBISE, McGill University

Isomonodromic deformations and Stability

One considers isomonodromic deformations of bundles with connection over a Riemann surface. When the underlying surface is a Riemann sphere with marked points, one has a nice link to representations into matrix groups when the bundle is trivial. One can ask whether generic deformations take one into this trivial locus. In other genera, one of the analogues of triviality of the bundles in genus zero is the notion of stability; we examine how deformations and stability interact. (Joint with Indranil Biswas and Viktoria Heu.)

Yael Karshon, University of Toronto

The Morse-Bott-Kirwan condition is local

Kirwan identified a condition on a smooth function, being "minimally degenerate", under which the usual techniques of Morse–Bott theory can be applied to the function. We prove that if a function satisfies this condition locally then it also satisfies the condition globally. As a corollary, we obtain a Morse-Lemma-type characterization of minimally degenerate functions. As another application, we use the local normal form theorem to recover Kirwan's result that the norm-square of a momentum map satisfies Kirwan's condition. This is joint work with Tara Holm.

LIAT KESSLER, University of Haifa

Homologically trivial symplectic cyclic actions need not extend to Hamiltonian circle actions

We give an example of a symplectic action of a cyclic group, inducing a trivial action on the homology, on a four manifold that admits Hamiltonian circle actions, and show that it does not extend to a Hamiltonian circle action.

This is a joint work with River Chiang.

DEREK KREPSKI, University of Manitoba

Prequantization of moment map theories

In 2003, P. Xu introduced quasi-symplectic groupoids $\Gamma_1 \rightrightarrows \Gamma_0$ as a natural target for moment maps, providing a unifying framework for various moment map theories. These include (classical) Hamiltonian actions of compact Lie groups, group-valued moment maps, Poisson-Lie group actions, and Hamiltonian loop-group actions. Quasi-symplectic groupoids are also known as twisted symplectic groupoids, which are the global objects integrating twisted Dirac structures. The 'twist' is encoded by a closed 3-form on Γ_0 (e.g. for group-valued moment maps, it is the Cartan 3-form on a compact Lie group $\Gamma_0 = G$).

This talk discusses prequantization in this framework. We briefly recall the prequantization of C. Laurent-Gengoux and P. Xu for the case of exact quasi-symplectic groupoids (where the twisting 3-form is exact), and discuss recent work that extends the definition of prequantization to non-exact quasi-symplectic groupoids.

JEREMY LANE, University of Toronto

Convexity For Momentum Maps Constructed by Thimm's Trick

In 1983, Guillemin and Sternberg introduced the Gelfand-Zeitlin system on coadjoint orbits and proved that the image of a Gelfand-Zeitlin system on a unitary coadjoint orbit is a convex polytope. It is interesting to note that although unitary coadjoint orbits are compact, the Gelfand-Zeitlin system only generates a Hamiltonian torus action on an open dense submanifold of the coadjoint orbit. For this reason, Guillemin and Sternberg were unable to deduce convexity and fibre connectedness for Gelfand-Zeitlin systems from their famous Abelian convexity theorem* (instead they computed the image explicitly).

The Gelfand-Zeitlin system can be viewed as an example of a more general construction, which one may refer to as 'Thimm's trick' (owing to a 1981 paper by Thimm). In this talk I will present a convexity and fibre connectedness theorem for all proper momentum maps that are constructed by Thimm's trick. It may be interesting for session participants to note that the proof does not contain Morse theory.

The corresponding pre-print is arXiv:1509.07356.

* Of course, the Abelian convexity theorem was proven by Atiyah and separately Guillemin and Sternberg in 1982.

YI LIN, Georgia Southern University

Hard Lefschetz property for contact manifolds

In the literature, there are two different versions of Hard Lefschetz theorems for a compact Sasakian manifold. The first version, due to Kacimi-Alaoui, asserts that the basic cohomology of a compact Sasakian manifold satisfies the Hard Lefschetz property. The second version, established more recently by Cappelletti-Montano, De Nicola, and Yudin, holds for the usual De Rham cohomology of a compact Sasakian manifold. In this talk, we will discuss a new approach to the Hard Lefschetz property of Sasakian manifolds, using the formalism of odd dimensional symplectic geometry. It leads to a Hard Lefschetz theorem for

the more general K-contact manifolds, which immediately implies that the two existing versions of Hard Lefschetz theorem are logically equivalent to each other. Our method sheds new insights on the topology of a Sasakian manifold. For instance, we will discuss how to use it to construct simply-connected K-contact manifolds which do not support any Sasakian structures in any dimension greater than or equal to seven. This in particular answers an open question asked by Boyer and late Galicki. If time permits, we will also discuss recently discovered topological obstructions to the existence of a Sasakian metric.

YIANNIS LOIZIDES, University of Toronto
Norm-square localization for Hamiltonian LG-spaces

Let $\psi : M \rightarrow L\mathfrak{g}^*$ be a proper Hamiltonian LG-space. Dividing out by the free action of the based loop group L_0G , yields a finite dimensional quasi-Hamiltonian space $\Phi : M \rightarrow G$. It is possible to define twisted Duistermaat-Heckman (DH) measures for (M, Φ) , which are distributions on G encoding cohomological pairings on reduced spaces. Similar to the Hamiltonian case, these can be computed using abelian localization. We will discuss an analogue of norm-square localization—as initiated by Witten and studied in depth by Paradan—for twisted DH-measures of quasi-Hamiltonian spaces, where the terms of the formula are indexed by the components of the critical set of $\|\psi\|^2$.

KEVIN LUK, University of Toronto
Meromorphic line bundles

In usual complex algebraic geometry, the Picard group of holomorphic line bundles on a complex algebraic variety X is given by $H^1(\mathcal{O}_X^\times)$. In this talk, we will discuss about a meromorphic modification of this sheaf for which the first cohomology will represent the group of meromorphic line bundles on X with singularities along a divisor D in X . We will then give examples of these meromorphic line bundles and discuss how ideas from mixed Hodge theory are used to study these geometric objects. Using an example involving the complement of an elliptic curve in \mathbb{P}^2 , we will discuss how these meromorphic line bundles relate to open Calabi-Yau mirror symmetry. Time permitting, we will discuss the possible relationship of these meromorphic line bundles to regulators in arithmetic geometry.

ECKHARD MEINRENKEN, University of Toronto
Convexity for twisted conjugation

I will describe a convexity theorem for products of twisted conjugacy classes. The proof uses twisted-equivariant group-valued moment maps.

RUXANDRA MORARU, University of Waterloo
Moduli spaces of stable generalized holomorphic bundles on generalized Kähler manifolds

Generalized holomorphic bundles are the analogues of holomorphic vector bundles in the generalized geometry setting. For some generalized complex structures, these bundles correspond to co-Higgs bundles, flat bundles or Poisson modules. In this talk, I will discuss the deformation theory of stable generalized holomorphic bundles on generalized Kähler manifolds. I will also give explicit examples and explain how some of them are related to Poisson modules. This is work in progress with Shengda Hu.

STEVEN RAYAN, University of Toronto
The bottom of the nilpotent cone of twisted Higgs bundles on $\mathbb{C}P^1$

We study the minimal component of the critical set for a Morse-Bott function corresponding to the natural S^1 -action on the moduli space of stable twisted Higgs bundles over the Riemann sphere.

FRÉDÉRIC ROCHON, UQÀM

Renormalized volume on the Teichmüller space of punctured Riemann surfaces

We define and study the renormalized volume for geometrically finite hyperbolic 3-manifolds that may have rank-1 cusps. We prove a variation formula, and show that for certain families of convex co-compact hyperbolic metrics degenerating to a geometrically finite hyperbolic metric with rank-1 cusps, the renormalized volume converges to the renormalized volume of the limiting metric. This is a joint work with Colin Guillarmou and Sergiu Moroianu.

REYER SJAMAAR, Cornell University

Induction of representations and Poincare duality

In 1965 Bott enumerated all possible ways to induce representations of compact Lie groups by means of elliptic differential operators. This led to generalizations of the Weyl character formula. I will review and update Bott's work and discuss some applications to K-theory. This is a report on joint work with Greg Landweber.

JORDAN WATTS, University of Colorado at Boulder

Tame Circle Actions

A famous question of Dusa McDuff, often referred to as the "McDuff Conjecture", is whether there exists a non-Hamiltonian symplectic circle action with isolated fixed points on a compact symplectic manifold. Susan Tolman recently answered this question in the affirmative, constructing a 6-dimensional such space with exactly 32 fixed points. A crucial ingredient to this construction involves Hamiltonian circle actions on complex manifolds and orbifolds in which the interaction between the complex structure and the symplectic form is fairly weak. Specifically, versions of Sjamaar's holomorphic slice theorem, the birational equivalence theorem of Guillemin and Sternberg, as well as reduction, cutting, and blow-up (all of which work in the Kaehler world) are required in this weaker setting.

All of these theorems and constructions are extended to this weaker setting in joint work by Tolman and myself. In particular, by weak, we mean that if ξ is the vector field induced by the circle action, ω the symplectic form, and J the complex structure, then $\omega(\xi, J\xi) > 0$ on the complement of the fixed point set. This condition is sufficient for all of the theorems and constructions above except for the blow-up (which also requires tameness at the point to be blown-up).

In this talk, I will focus on the holomorphic slice theorem about a fixed point, reduction, and (time-permitting) the birational equivalence theorem proven in the joint paper.

JONATHAN WEITSMAN, Northeastern University

On b-symplectic manifolds

We describe recent progress on b-symplectic manifolds. (Joint work with Victor Guillemin and Eva Miranda)