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Moduli spaces of real vector bundles over a real curve

Given a Riemann surface X, we may associate a moduli space $M(X)$ of stable holomorphic vector bundles of some fixed rank and degree. If the rank and degree are coprime, then $M(X)$ is a compact Kaehler manifold. Atiyah and Bott showed how $M(X)$ can be constructed as an infinite dimensional symplectic quotient, and used this to compute Betti numbers and other topological information.

Suppose now that X comes equipped with an anti-holomorphic involution τ. This induces an involution of $M(X)$ and the fixed point set, $M(X,\tau)$, is a real Lagrangian submanifold of $M(X)$. Biswas-Huisman-Hurtubise and Schaffhauser showed how $M(X,\tau)$ can be understood as a moduli space of real vector bundles over (X,τ) and can be constructed as an infinite dimensional "real Lagrangian quotient". In this talk, I will explain how the methods of Atiyah and Bott can be adapted to compute the Z_2-Betti numbers of $M(X,\tau)$. I will also comment on how these $M(X,\tau)$ form a promising class of examples for Lagrangian Floer theory.