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Tame Circle Actions

A famous question of Dusa McDuff, often referred to as the "McDuff Conjecture", is whether there exists a non-Hamiltonian symplectic circle action with isolated fixed points on a compact symplectic manifold. Susan Tolman recently answered this question in the affirmative, constructing a 6-dimensional such space with exactly 32 fixed points. A crucial ingredient to this construction involves Hamiltonian circle actions on complex manifolds and orbifolds in which the interaction between the complex structure and the symplectic form is fairly weak. Specifically, versions of Sjamaar's holomorphic slice theorem, the birational equivalence theorem of Guillemin and Sternberg, as well as reduction, cutting, and blow-up (all of which work in the Kaehler world) are required in this weaker setting.

All of these theorems and constructions are extended to this weaker setting in joint work by Tolman and myself. In particular, by weak, we mean that if ξ is the vector field induced by the circle action, ω the symplectic form, and J the complex structure, then $\omega(\xi, J\xi) > 0$ on the complement of the fixed point set. This condition is sufficient for all of the theorems and constructions above except for the blow-up (which also requires tameness at the point to be blown-up).

In this talk, I will focus on the holomorphic slice theorem about a fixed point, reduction, and (time-permitting) the birational equivalence theorem proven in the joint paper.