Mathematical Finance
Finance mathématique
(Org: **Alexandru Badescu** (Calgary), **Cody Hyndman** (Concordia) and/et **Alexandre Roch** (UQAM))

ETIENNE CHEVALIER, Université d'Evry
Indifference fee rate for variable annuities

In this paper, we work on indifference valuation of variable annuities and give a computation method for indifference fees. We focus on the guaranteed minimum death benefits and the guaranteed minimum living benefits and allow the policyholder to make withdrawals. We assume that the fees are continuously payed and that the fee rate is fixed at the beginning of the contract. Following indifference pricing theory, we define indifference fee rate for the insurer as a solution of an equation involving two stochastic control problems. Relating these problems to backward stochastic differential equations with jumps, we provide a verification theorem and give the optimal strategies associated to our control problems. From these, we derive a computation method to get indifference fee rates. We conclude our work with numerical illustrations of indifference fees sensibilities with respect to parameters.

CHRISTOPH FREI, University of Alberta
Systemic Influences on Optimal Equity-Credit Investment

We introduce an equity-credit portfolio framework taking into account the structural interaction of market and credit risk, along with their systemic dependencies. We derive a closed-form expression for the optimal investment strategy in stocks and credit default swaps (CDSs). We exploit its representation structure and analyze the mechanisms driving the optimal investment decisions. The transmission of market risk premia is the key mechanism through which systemic influences affect the optimal investment strategy. We develop a novel calibration procedure and find that systemic dependencies are statistically significant when the model is fitted to historical time series of equity and CDS data. An empirical analysis with data of companies in the DJIA reveals the critical role of systemic risk in portfolio monitoring. The talk is based on joint work with Agostino Capponi (Columbia University).

HUAN GEORGIA GAO, Bank of Montreal
Risk Measurement of a Guaranteed Annuity Option under a Stochastic Environment

We address the problem of setting capital reserves for a guaranteed annuity option (GAO). The modelling framework for the loss function of GAO is developed. A one-decrement actuarial model is considered in which death is the only decrement, and the interest and mortality risk factors follow correlated affine structures. Risk measures are determined using moment-based density method and benchmarked with the Monte-Carlo simulation. Bootstrap technique is utilised to assess the variability of risk measure estimates. We establish the relation between a desired level of risk measure accuracy and required sample size under the constraints of computing time and memory. A sensitivity analysis of parameters is further conducted, and our numerical investigations provide practical considerations for insurers in meeting certain regulatory requirements.

ANDREW HEUNIS, University of Waterloo
Utility maximization in a regime switching model with convex portfolio constraints and margin requirements

We study a problem of stochastic control in mathematical finance, with the goal of maximizing expected utility of investment and consumption over a finite trading horizon. The asset prices are modeled by Ito processes, for which the market parameters are subject to *regime switching* in the sense of being adapted to the joint filtration of the driving Brownian motion and a finite-state Markov chain which models "regime states" of the market. The vector of portfolios is constrained to a specified closed and convex set, and margin payments are levied on the investor, resulting in a wealth equation which is *nonlinear* in the portfolio. We proceed by the method of *conjugate duality* to construct a *dual optimization problem* together with *optimality*

relations between putative solutions of the given (i.e."primal") optimization problem and the dual optimization problem. These optimality relations are then used to address the specific cases of *power-type* and *logarithmic* utility functions, with *convex cone* portfolio constraints, and a higher rate of interest for borrowing than for lending. We get completely explicit optimal portfolios and characterize the optimal consumption rate as the solution of a backward stochastic differential equation (BSDE) "driven" by the canonical martingales of the regime-state Markov chain. For the power utility function this is a rather unconventional BSDE, to which standard existence results do not apply, and accordingly we establish existence and uniqueness of solutions for this BSDE.

REG KULPERGER, University of Western Ontario
Nonlinear Time Series Models in Mathematical Finance

In mathematical finance both continuous and discrete time models are used. Continuous time models, in particular geometric Brownian motion (GBM), are easy to implement and use. However they have certain assumptions. The log of GBM is Brownian motion with drift and so has independent Gaussian increments and is Markov. Discrete time series models may still have conditional Gaussian or other driving noise, but might not be Markov. However they are not as simple to implement, for option pricing or hedging etc. In this talk we consider GARCH models, which are non Markov except for ARCH(1). In particular we compare some properties of GARCH in mean and ARMA GARCH for options.

If time permits we some non linear multivariate time series models.

The main part of this work was done with a recent student Yi Xi. Some other parts of this work were done with Hao Yu, Alex Badescu, Weibin Jiang and Zi Zhen Liu.

CHANTAL LABBÉ, HEC Montréal
Aumann-Shapley allocation of incremental CVA

The importance of accounting for counterparty risk in the evaluation of a portfolio is widely recognized, especially since the 2007 financial crisis. The credit value adjustment (CVA) is thus computed, and at the counterparty level to capture the effect of netting all of the portfolio components. Internal management issues then raise a question: how to allocate the global CVA to the individual components of the portfolio? A common approach is to allocate a trade the incremental CVA at the time it is initiated, i.e. the difference between the CVA of the portfolio when including and excluding this trade. However, this approach is only applicable in institutions where CVA is computed in real time, leaving out most. For discrete time, the incremental CVA remains a natural tool to isolate the contribution of the trades initiated since last CVA evaluation date. This work examines the problem of fairly allocating this joint contribution to each of the new trades. This problem shares much similarities with the well known risk capital allocation problem, where the goal is to allocate the risk capital among divisions of the firm, and for which the Aumann-Shapley allocation principle is suitable. The literature on risk capital allocation typically considers risk measures satisfying nice properties. Even if such nice properties appear desirable, industry finds value in risk measures not satisfying them, such as CVA. We show how the Aumann-Shapley principle is nevertheless amenable to the incremental CVA allocation problem.

VATHANA LY VATH, ENSIIE/Université d'Evry
Liquidity risk and optimal dividend/investment strategies

In this paper, we study the problem of determining an optimal control on the dividend and investment policy of a firm operating under uncertain environment and risk constraints. We allow the company to make investment decisions by acquiring or selling productive assets whose value is governed by a stochastic process. The firm may face liquidity costs when it decides to buy or sell assets. We formulate this problem as a multi-dimensional mixed singular and multi-switching control problem and use a viscosity solution approach. We numerically compute our optimal strategies and enrich our studies with numerical results and illustrations.

ROMAN MAKAROV, Wilfrid Laurier University
Modelling Liquidation Risk with Occupation Times

In this talk, we develop a structural model for the liquidation risk of a firm subject to both Chapters 7 and 11 of the U.S. bankruptcy code. The firm's value evolves as geometric Brownian motion. Liquidation is triggered by one of the following two events: (1) the firm's asset value reaches the liquidation barrier and (2) the value process cumulatively stays below the default barrier over a grace period. The main contribution of this talk is the development of analytical formulae for Laplace transforms of occupation time distributions of a drifted Brownian motion with an absorbing barrier. As a result, we develop a semi-analytic formula, easily implemented via quadrature, for the probability of liquidation. In this talk, we also discuss pricing of the firm's debt.

ALEXANDER MELNIKOV, University of Alberta
Partial hedging for defaultable securities and its connection with equity-linked life insurance

The talk is devoted to partial hedging for defaultable markets with multiple default times. Our attention is focused on efficient hedging problem. We give its solution for a "defaultable" Black-Scholes model using the fundamental Neumann-Pearson lemma and non-smooth convex duality. In case of zero recovery rates we find a closed form solution. It is shown how to exploit these results in pricing of equity-linked life insurance contracts.

ADAM METZLER, Wilfrid Laurier University
Regulatory Discretion and the Valuation of CoCo Bonds

A contingent convertible (CoCo) bond begins life as subordinated debt, but converts into common equity when the issuing institution begins to experience financial distress. In practice conversion triggers involves both an objective component (e.g. a CET1 ratio below a fixed threshold such as 5.5%) and a subjective component (e.g. regulatory judgement that the firm is non-viable), however the nascent academic literature on CoCo valuation tends to ignores the subjective component. In this talk we develop a model (to the best of our knowledge, the first model) that explicitly accounts for regulatory discretion in the conversion trigger. We assume that conversion occurs at the first event time of a Cox process whose intensity is a function of the firm's asset value (which evolves as affine geometric Brownian motion), with the functional form reflecting investors' beliefs on how the regulator is likely to act. Bond valuation reduces to computing conditioned Laplace transforms in the context of killed diffusions (e.g. the transform of the killing time, conditioned on the process remaining above a prespecified level prior to killing), each of which can be characterized as the solution to an appropriate ODE. Time permitting we will discuss numerical results when the model is calibrated to balance-sheet data fro Canadian banks.

MOBOLAJI OGUNSOLU, University of Calgary
Credit risk pricing via Epstein-Zin pricing kernel

We present an equilibrium framework for credit risk pricing based on the Epstein-Zin preferences where the default time of the firm is modeled as the first hitting time of a default barrier by an unobservable process, representing the firm's value. Our goal is to extend the results of Duffie and Lando on term structures of credit yield spreads under incomplete accounting information to a setting where investors' preferences are also taken into account. The observed variables are the state variables, aggregate consumption and volatility, and the default indicator process. The drift of the firm's value is assumed to be a function of aggregate consumption. The state variables and the firm 's value are modeled as affine diffusion processes. Using the framework of Eraker and Shaliastovich (2008), we obtain the dynamics of our system with respect to an equivalent equilibrium pricing measure. In a specific example , the price of a zero-coupon bond is expressed in terms of a solution of a system of second order parabolic partial differential equation (PDE) which is solved via numerical techiques Finally, the joint implications of investor's preferences and imperfect information on the credit yield spreads are analyzed.

Joint work with Deniz Sezer(UCalgary)

SERGIO PULIDO, ENSIIE / Université d'Évry Val d'Essonne
The Jacobi stochastic volatility model

We introduce a novel stochastic volatility model, where the squared volatility could be bounded and follows a Jacobi process. This model comprises the Heston model as a limiting case. The price of a European call option admits a closed form series representation. This representation involves the moments of the log price of the asset, which are given in terms of a linear ordinary differential equation. We demonstrate that the numerical computations are robust and perform particularly well. In addition, we present theoretical bounds for the error in the price approximation. The proposed pricing method has important implications as it could be applied to a wide range of European type claims. This is joint work with Damien Ackerer and Damir Filipovic.

ALEXEY RUBTSOV, Ryerson University
Robust portfolio choice with derivative trading under stochastic volatility

We determine the optimal portfolio for an ambiguity averse investor who has access to stock and derivatives markets. The stock price follows a stochastic volatility jump-diffusion process and the investor can have different levels of uncertainty about the diffusion parts of the stock and its volatility. We find strong evidence that the optimal exposures to stock and volatility risks are significantly affected by the ambiguity aversion to the corresponding risk factor only. We also show that volatility ambiguity has a smaller impact in incomplete markets. Investors who ignore jump risk/model uncertainty/derivatives always incur welfare losses. In our numerical example, the loss from neglecting model uncertainty can be almost as much as the loss from not trading the derivatives.

TOM SALISBURY, York University
Optimal tontines

Annuities are the traditional product used to guarantee sustainable income in retirement. But because they expose the issuer to systematic longevity risk (the risk that the population will live longer than expected), annuitants are subject to both risk charges and solvency risk. Tontines are one of several products that have been proposed to address this issue, by protecting individuals against their idiosyncratic risk, without exposing issuers to systematic risk. Such a scheme might involve an investment pool, whose proceeds are shared only among survivors. We'll discuss the optimal design of such tontines, and will show how they can be adapted to handle a heterogeneous pool of investors. This is joint work with Moshe Milevsky (York University)

SIMONE SCOTTI, University Paris Diderot
Optimal Investment in Markets with Over and Under-Reaction to Information

In this paper we introduce a new jump-diffusion model for stock prices, which takes into account over and under-reaction of the market to incoming news. The jumps' impact on the assets dynamics is twofold: on one hand we use a Poisson process as a driver to obtain discontinuous trajectories and on the other hand the presence of jumps in the drift, via a shot noise process, allows to incorporate "fade-away" effects, meaning that the effects of these abrupt changes fade away as time goes by. Our model is a partial information one: the drift direction after a jump is not accessible to standard investors immediately after the jump.

We focus on a maximization of expected utility from terminal wealth problem, providing, in a logarithmic utility setting, the optimal investment strategy in explicit form, both under full (i.e., from the insider point of view) and under partial information (i.e., from the standard investor viewpoint). We test our results on real market data relative to Enron and Ahold.

The three main contributions of this paper are: the introduction of a new market model dealing with over and under-reaction to news, the explicit computation of the optimal filter dynamics using an approach based on enlargement of filtrations and the application of the optimal portfolio allocation rule to real market data in both full and partial information setting.

MIHAI SIRBU, The University of Texas at Austin
On modeling and analysis of continuous-time stochastic games

We present a new look at the modeling and dynamic programming analysis of zero-sum stochastic differential games. We consider both symmetric and non-symmetric strong formulations of games and analyze their relation with control problems under model uncertainty.

STEPHAN STURM, WPI (Worcester Poytechnic Institute)
Cost effciency in incomplete markets

We consider a portfolio choice problem maximizing terminal wealth on a finite time horizon. The cost-effciency in complete market characterizes the optimal portfolio as the minimizer of the risk neutral hedging costs among all random variables having the same distribution. We provide a generalization of the cost effciency principle to incomplete markets and provide applications to the characterization of the rationality of portfolio choice. This is joint work with Carole Bernard (Grenoble).

ANATOLIY SWISHCHUK, University of Calgary
A Semi-Markovian Modeling of Limit Order Markets

R. Cont and A. de Larrard (SIAM J. in Financial Mathematics, 2013) introduced a tractable stochastic model for the dynamics of a limit order book, computing various quantities of interest such as the probability of a price increase or the diffusion limit of the price process. As suggested by empirical observations, we extend in this talk their framework to 1) arbitrary distributions for book events inter-arrival times (possibly non-exponential) and 2) both the nature of a new book event and its corresponding inter-arrival time depend on the nature of the previous book event. We do so by resorting to Markov renewal processes to model the dynamics of the bid and ask queues. We keep analytical tractability via explicit expressions for the Laplace transforms of various quantities of interest. We justify and illustrate our approach by calibrating our model to the five stocks Amazon, Apple, Google, Intel and Microsoft on June 21st 2012. As in Cont and Larrard, the bid-ask spread remains constant equal to one tick, only the bid and ask queues are modeled (they are independent from each other and get reinitialized after a price change), and all orders have the same size. (This talk is based on our joint paper with Nelson Vadori (Morgan Stanley)).

RENJIE WANG, Concordia University
Optimal measure transformation problems

We introduce an optimal measure transformation problem for zero coupon bond prices based on dynamic relative entropy of probability measures. In the default-free case we prove the equivalence of the optimal measure transformation problem and an optimal stochastic control problem of Gombani and Runggaldier (Math. Financ. 23(4):659-686, 2013) for bond prices. We also consider the optimal measure transformation problem for defaultable bonds, futures contracts, and forward contracts. We provide financial interpretations of the optimal measure transformation problems in terms of the maximization of returns subject to a relative entropy penalty term. In general the solution of the optimal measure transformation problem is characterized by the solution of certain decoupled nonlinear forward-backward stochastic differential equations (FBSDEs). In specific classes of models we show how these FBSDEs can be solved explicitly or at least numerically.